Fundamenta Informaticae 43(2001) 2–32
IOS Press

Qualitative Spatial Representation and Reasoning : An Overview

A G Cohn **S M Hazarika**

School of Computing
University of Leeds, Leeds LS2 9JT
United Kingdom
e-mail:{agc,smh}@comp.leeds.ac.uk

Abstract. The paper is a overview of the major *qualitative spatial representation* and *reasoning techniques*. We survey the main aspects of the representation of qualitative knowledge including ontological aspects, topology, distance, orientation and shape. We also consider *qualitative spatial reasoning* including reasoning about spatial change. Finally there is a discussion of theoretical results and a glimpse of future work. The paper is a revised and condensed version of [33, 34].

Keywords: Qualitative Spatial Reasoning, Ontology.

*The text is in a slightly different format from the FI format.

1. Introduction

Qualitative Reasoning is concerned not only with capturing the everyday common-sense knowledge of the physical world, but also the myriad equations used by engineers and scientists to explain complex physical phenomenon while creating quantitative models [180]. The principal goal of Qualitative Reasoning is to make explicit this knowledge, so that given appropriate reasoning techniques, a computer could make predictions, diagnose and explain the behaviour of physical systems in a qualitative manner[1] without recourse to an often intractable or perhaps unavailable quantitative model.

The essence of qualitative reasoning is to find ways to represent continuous properties of the world by discrete systems of symbols. One can always quantize something continuous, but not all quantizations are equally useful. One way to state the idea is the relevance principle: the distinctions made by a quantization must be relevant to the kind of reasoning performed [76]. The resulting set of qualitative values is termed a quantity space, in which indistinguishable values have been identified into an equivalence class[2]. There is normally a natural ordering (either partial or total) associated with a quantity space, and one form of simple but effective inference is to exploit the transitivity of the ordering relation; another is to devise qualitative arithmetic algebras [180]; typically, these may produce ambiguous answers. Much research in the Qualitative Reasoning literature is devoted to overcoming the detrimental effects on the search space resulting from this ambiguity.

Another important aspect of work in traditional qualitative reasoning worth noting here is the standard assumption that change is continuous. A simple consequence is that while changing, a quantity must pass through all the intermediate values. For example, in the frequently used quantity space $\{-, 0, +\}$, a variable cannot transition from '$-$' to '$+$' without first going through value 0. This notion is also exploited in the work on qualitative spatial reasoning.

2. What is Qualitative Spatial Reasoning ?

Spatial reasoning, in our every day interaction with the physical world, in most cases is driven by qualitative abstractions rather than complete a priori quantitative knowledge. Therefore, QR holds promise for developing theories[3] for reasoning about space. This justifies the increasing interest in the study of spatial concepts from a cognitive point of view which provoked the birth

[1]Note that although one use for qualitative reasoning is that it allows inferences to be made in absence of complete knowledge, it does this not by probabilistic or fuzzy techniques (which may rely on arbitrarily assigned probabilities or membership values) but by refusing to differentiate between quantities unless there is sufficient evidence to do so.

[2]The case where the indistinguishability relation is not an equivalence relation has not been much considered, except by [121, 116]. However, an important characteristic of perceptual acuity is that a series of small changes, each imperceptible, may combine to form a perceptible change. Then the indistinguishability relation though reflexive and symmetric, is not transitive, which can lead to a paradox of perception. Placing a non-transitive theory of indistinguishability within a possible-worlds theory of perception and knowledge, [48] presents ways of avoiding the paradox.

[3]The word theory is used in its logical/mathematical context i.e., a set of formal axioms which specify the properties and relations of a collections of entities, not in the natural scientist's sense of an empirically testable explanation of observed regularities.

of Qualitative Spatial Reasoning (QSR) within AI and also GIS.

Space being multidimensional, is not adequately represented by a single scalar quantity. The multidimensional nature of space has been ill-addressed in QR despite some early forays such as Hayes' Naive Physics Manifesto [111, 77]. However, this neglect can be partially attributed to the 'poverty conjecture' [180]. Although purely qualitative representations were reasonably successful in reasoning about many physical systems [180], there was much less success in developing purely qualitative reasoners about spatial and kinematic mechanisms and the poverty conjecture [77] is that this is in fact impossible – there is no purely qualitative spatial reasoning mechanism. Forbus et al. correctly identify transitivity of values as a key feature of qualitative quantity spaces but doubt that this can be exploited much in higher dimensions and conclude that the space of representations in higher dimensions is sparse and for spatial reasoning nothing weaker than numbers will do.

The challenge of QSR then is to provide calculi which allow a machine to represent and reason with spatial entities without resort to the traditional quantitative techniques prevalent in, for e.g. the computer graphics or computer vision communities.

There has been an increasing amount of research over the last couple of years which tends to refute, or at least weaken the 'poverty conjecture'. Qualitative spatial representations addressing many different aspects of space including topology, orientation, shape, size and distance have been put forward. There is a rich diversity of these representations and they exploit the transitivity as demonstrated by the relatively sparse composition tables (cf the well known table for Allen's interval temporal logic [180]) which have been built for these representations.

This paper is an overview of the major qualitative spatial representation and reasoning techniques. In section 3 we will mention some possible applications of qualitative spatial reasoning. Thereafter, in section 4 we survey the main aspects of the representation of qualitative spatial knowledge including ontological aspects, topology, distance, orientation and shape. The next section discusses qualitative spatial reasoning including reasoning about spatial change. Finally, the paper concludes with a presentation of theoretical results and a glimpse at future work. The paper is a revised and condensed version of [33, 34].

3. Applications of Qualitative Spatial Reasoning

Research in QSR is motivated by a wide variety of possible application areas including Geographic Information System (GIS), robotic navigation, high level vision, spatial propositional semantics of natural languages, engineering design, common-sense reasoning about physical systems and specifying visual language syntax and semantics. There are numerous other application areas including qualitative document-structure recognition [81], the notion of a nîche (e.g. in biology [168]) and domains where space is used as a metaphor, e.g. [125, 147].

Even though GIS are now a commonplace, the major problem is that of interaction. With gigabytes of information stored either in vector or raster format, present-day GISs do not sufficiently support intuitive or common-sense oriented human-computer interaction. Users may wish to abstract away from the mass of numerical data and specify a query in a way which is essentially, or at least largely, qualitative. Arguably, the next generation GIS will be built on concepts arising from *Naive Geography* [65]. Much of naive geography should employ qualitative reasoning techniques, perhaps combined with the provision of "spatial query by sketch"[59].

Although robotic navigation ultimately requires numerically specified directions to the robot to move or turn, hierarchical planning with detailed decisions (e.g. how or exactly where to move) being delayed until a high level plan have been achieved has been shown to be effective [172]. Further, the robot's model of its environment may be imperfect, leading to an inability to use standard robot navigation techniques. Under such circumstances, a qualitative model of space may facilitate planning. One such approach is the development of a robust qualitative method for robot exploration, mapping and navigation in large-scale spatial environments described in [124]. A qualitative solution to the well known 'piano mover's problem' is [72].

QSR has been used in computer vision for visual object recognition at a higher level which includes the interpretation and integration of visual information. Using qualitative symbolic projections, it is possible to reason about the shape of objects to match geographical objects. QSR technique have been used to interpret the results of low-level computations as higher level descriptions of the scene or video input [73]. The use of qualitative predicates helps to ensure that scenes which are semantically close have identical or at least very similar descriptions.

In natural language, the use and interpretation of spatial propositions tend to be ambiguous. There are multiple ways in which natural language spatial prepositions can be used (e.g. [115] cites many different meanings of "in"); herein lies the motivation of qualitative spatial representation for finding some formal way of describing these prepositions (e.g. [4, 159])

Engineering design, like robotic navigation, ultimately normally requires a fully metric description. However, at the early stages of the design process, a reasonable qualitative description would suffice. The field of qualitative kinematics (e.g. [71]) is largely concerned with supporting this type of activity.

Finally, visual languages, either visual programming languages or some kind of representation language, lack a formal specification of the kind that is normally expected of a textual programming or representation language. Although some of these languages make metric distinctions, the bulk of it is often predominantly qualitative in the sense that the exact shape, size, length etc. of the various components of the diagram or picture is unimportant – rather, what is important is the topological relationship between these components. Therein lies the promise of a theory of qualitative spatial reasoning in specifying such languages [95, 107].

4. Aspects of Qualitative Spatial Representation

Representing space has a rich history in the physical sciences – and serves to locate objects in a quantitative framework. At the other extreme, spatial expressions tend to operate on a loose partitioning of the domain. Representation for this less precise description of space proliferated, more or less on an *ad hoc* basis until the emergence of qualitative spatial reasoning whereafter the partitioning was done more systematically [135].

There are many different aspects to space and therefore to its representation. Not only do we have to decide on what kind of spatial entity we will admit (i.e. commit to a particular ontology of space), but also we can consider developing different kinds of ways of describing the relationship between these kinds of spatial entities; for example we may consider just their topology, or their sizes or the distance between them, their relative orientation or their shape.

4.1. Ontology

Traditionally, in mathematical theories of space, points are considered as primary primitive spatial entities (or perhaps points and lines), and extended spatial entities such as regions are defined, if necessary, as sets of points. A minority tradition ('mereology' or 'calculus of individuals') regards this as a philosophical error[4]. Within the QSR community, there is a strong tendency to take regions of space as the primitive spatial entity – see [177]. Even though this ontological shift means building new theories for most spatial and geometrical concepts, there are strong reasons for taking regions as the ontological primitive. If one is interested in using the spatial theory for reasoning about physical objects, then one might argue that the spatial extension of any physical object must be region-like rather than a lower dimension entity. Further, one can always define points, if required, in terms of regions [13]. However, it needs to be admitted that at times it is advantageous to view a 3D physical entity as a 2D or even a 1D entity. Of course, once entities of various dimensions are admitted, a pertinent question would be whether mixed dimension entities are allowed. Further discussion of this issue can be found in [41, 35, 103, 143]

Another ontological question is what is the nature of the embedding space, i.e. the universal spatial entity? Conventionally, one might take this to be R^n for some n, but one can imagine applications where discrete (e.g. [66]), finite (e.g. [102]), or non convex (e.g. non connected) universes might be useful. There is a tension between the continuous-space models favoured by high-level approaches to handling spatial information and discrete, digital representations used at the lower level. An attempt to bridge this gap by developing a high-level qualitative spatial theory based on a discrete model of space is [88]. For a recent investigation into discrete vs continuous space, see [133].

Once one has decided on these ontological questions, there are further issues: in particular, what primitive "computations" should be allowed? In a logical theory, this amounts to deciding what primitive non logical symbols one will admit without definition, only being constrained by some set of axioms. One could argue that this set of primitives should be small, not only for mathematical elegance and to make it easier to assess the consistency of the theory, but also because this will simplify the interface of the symbolic system to a perpetual component because fewer primitives have to be implemented. The converse argument might be that the resulting symbolic inferences may be more complicated or that it is more natural to have a large and rich set of concepts which are given meaning by many axioms which connect them in many different ways [110].

One final ontological question is how to model the multi dimensionality of space? One approach is to model space by considering each dimension separately, projecting each region to each of the dimensions and reasoning along each dimension separately. However, this approach is grossly inadequate, as two objects overlap when projected on to both the x and y axes individually, when in fact they may not overlap at all and only useful in domains with rectangular, orthogonally aligned objects [179]

[4] Simons [164] says : "No one has ever perceived a point, or ever will do so, whereas people have perceived individuals of finite extent".

4.2. Topology

Topology is perhaps the most fundamental aspect of space. It is clear that topology must form a fundamental aspect of qualitative spatial reasoning since topology certainly can only make qualitative distinctions. Although topology has been studied extensively within the mathematical literature, much of it is too abstract to be of relevance to those attempting to formalise common-sense spatial reasoning. Although various qualitative spatial theories have been influenced by mathematical topology, there are number of reasons why such a wholesale importation seems undesirable in general [103].

Moreover, we are interested in qualitative spatial reasoning and not just representation, and this has been paid little attention in mathematics. Neither point-set nor algebraic topology is particularly well-adapted to reasoning of the forms such as: Given that a region a is in relation R_1 to region b, and region b is in relation R_2 to region c; what relations may or must hold between a and c? Of course, it might be possible to adapt the conventional mathematical formalisms, and indeed this strategy has been adopted [60, 64, 184]. One existing approach to topology which has been espoused by the QSR community is the work to be found in the philosophical logic community [181, 51, 183, 23, 24, 13]. This work has built axiomatic theories of space which are predominantly topological in nature, and which take regions rather than points as primitive - indeed, this tradition has been termed as "pointless geometries" [90]. In particular the work of Clarke [23, 24] has lead to the development of the so called RCC systems [152, 151, 150, 148, 47, 36, 7, 98, 32, 103, 41, 35] and also has been developed further e.g. [176, 3].

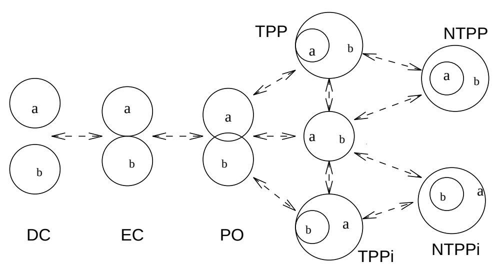

Figure 1 2D illustrations of the relations of RCC-8 calculus and their continuous transitions (*conceptual neighbourhood*).

Clarke took as his primitive notion the idea of two regions x and y being connected (sharing a point, if one wants to think of regions as consisting of sets of points): $\mathsf{C}(x,y)$. In the RCC system this interpretation is slightly changed to the closures of the regions sharing a point. Actually, given the disdain of the RCC theory as presented in [151] for points, a better interpretation, given some suitable distance metric, would be that $\mathsf{C}(x,y)$ means that the distance between x and y is zero, c.f. [170]. This has the effect of collapsing the distinction between a region, its closure and its interior, which it is argued has no relevance for the kinds of domain with which QSR is concerned. $\mathsf{C}(x,y)$ is surprisingly powerful: it is possible to define many predicates and functions which capture interesting and useful topological distinctions. The set of eight jointly exhaustive and pairwise disjoint (JEPD) relations illustrated in Figure 1 are one particularly useful set (often known as the RCC-8 calculus).

A formal semantics for RCC has been given by [99, 52, 170]. Furthermore, a canonical model for arbitrary ground Boolean wffs over RCC-8 atoms has been proposed [154] which is then utilised in a procedure to generate an actual 2D or 3D interpretation. Moreover this ensures (contrary to [3, 176]), that an individual is connected to its complement, and that the principle of supplementation [164] holds. The work of [176, 3] mentioned above is also based on Clarke's Calculus. The original interpretation of $\mathsf{C}(x, y)$ is retained, though the fusion operator is discarded, it is made first order and several errors are corrected. Contrary to the RCC interpretation, [3] argue that differentiating between an individual, its closure and its interior is cognitively important: material objects are closed individuals and their complements are open ones, so their interpretations do not share any points. A framework wherein the different notions of connection in the literature are analyzed is presented in [45, 44]. Three dimensions of variation are considered: whether or not the closure of a region is considered; whether the connection is point or line/surface-like; the degree of connection amongst the components of multipiece regions.

4.2.1. Expressiveness of $\mathsf{C}(x, y)$

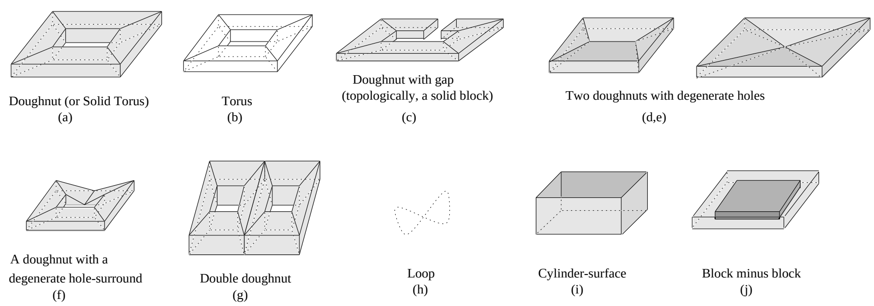

Figure 2. It is possible to distinguish all the above shapes using $\mathsf{C}(x,y)$ alone.

Taxonomies of topological properties and relations can be defined using the single predicate $\mathsf{C}(x, y)$. Apart from the simple RCC-8 relations, the predicate $\mathsf{C}(x, y)$ can be used to define many more predicates. For example one could define predicates which counted the number of times two regions touch. In a series of papers, [97, 98, 103, 101], Gotts sets himself the task of distinguishing a 'doughnut' (a solid one piece region with a single hole). It is shown how under a restrictive set of assumptions about the topological properties of the regions in general, and the target region in particular, all shapes depicted in Figure 2 can be distinguished.

Another range of topological distinctions between one-piece (CON) regions can be made (under certain assumptions) using C. As shown in Figure 3, a region, if it is connected, may or may not be interior connected (INCON); meaning that the interior of the region is all of one piece. It is relatively easy to express this property (or its converse) in RCC terms. However $\mathsf{INCON}(r)$ does not rule out all regions with anamolous boundaries, and in particular does not exclude the regions (d,e,f) of Figure 2, which do have one-piece interiors, but which nevertheless have boundaries which are not (respectively) simple curves or surfaces, having 'anomalies' in the form of points which do not have line-like (or disc-like) neighbourhoods within the boundary (i.e. which are locally Euclidean). It appears possible using $\mathsf{C}(x, y)$ to define a predicate (WCON)

that will rule out the anomalous cases of Figure 2, but it is by no means straightforward[5], and it is not demonstrated conclusively in [98] that the definitions do what is intended.

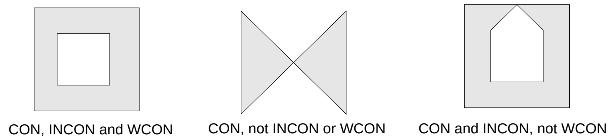

Figure 3. Different types of CON Region

One source of the difficulties arising is the fact that within RCC, since all regions in a particular model of the axioms are of the same dimensionality as the universal region, u, assuming u itself to be of uniform dimensionality (this follows from the fact that all regions have a NTTP), there is no way to refer directly to the boundary of a region or to the dimensionality of the shared boundary of two EC regions, or to any relations between entities of different dimensionalities. There has been a tendency in much of the work involving qualitative spatial reasoning to assume, if only implicitly, that the spatial entities considered in any one theory should have the same dimension. In cases where reasoning about dimensionality becomes important, RCC and related systems based on the $\mathsf{C}(x, y)$ predicate are not very powerful[6]. The INCH calculus [100] treats points and spatially extended entities as specializations of the more general notion of a 'spatial extent'. It aims to improve on the expressiveness of connection-based calculi such as RCC, while avoiding the counterintuitive consequences of a point-set approach.

Fleck argues that standard models of space and time based on segmenting R^n have the wrong topological structure at object, event or region boundaries [74]; and illustrates how to modify an R^n model to have an appropriate topological structure. Another proposal addressing the problem of representing and reasoning about regions of differing dimensionality (though still not of mixed dimensionality) is [85]. Using a variant of the Classical Extensional Mereology originating in the work of Lesniewski (as recounted in [164]), through use of the mereological part relation $\mathsf{P}(x, y)$ and a topological boundary primitive, $\mathsf{B}(x, y)$ – x is the boundary of y (x being a region of one dimension less), leads towards the desired intuitive picture of a strictly linear hierarchy of dimensions. Other theories which introduce the notion of boundaries of regions explicitly include [166, 175, 152, 167].

4.2.2. Topology via "n-intersections"

An alternative approach to representing and reasoning about topological relations has been promulgated via a series of papers [57, 60, 58, 26, 61, 64]. Three sets of points are associated with every region - its interior, boundary and complement. The relationship between any two region can be characterized by a 3x3 matrix[7] called the 9-intersection. Although it would seem

[5] Note, however, that this task becomes almost trivial once the $\mathsf{conv}(x)$ primitive is introduced - see Section 4.4.3.

[6] To reason about regions of different dimensionality is impossible without imposing a sort structure and essentially taking a copy of the theory for each dimension-sort.

[7] Actually, a simpler 2x2 matrix [60] known as the 4-intersection featuring just the interior and the boundary is sufficient to describe the eight RCC relations. However the 3x3 matrix allows more expressive sets of relations to be defined as noted below since it takes into account the relationship between the regions and its embedding space.

that there are $2^9 = 512$ possible matrices, after taking into account the physical reality of 2D space and some specific assumptions about the nature of regions, it turns out that the there are exactly 8 remaining matrices corresponding to the RCC-8 relations. One can use this calculus to reason about regions which have holes by classifying the relationship not only between each pair of regions, but also the relationship between each hole of each region and the other region and each of its holes [63].

Different calculi with more JEPD relations can be derived by changing the underlying assumptions about what a region is and by allowing the matrix to represent the codimension of intersection. For example, one may derive a calculus for representing and reasoning about regions in Z^2 rather than R^2 [66]. Alternatively, one can extend the representation in each matrix cell by the dimension of the intersection rather than simply whether it exists or not [27]. This allows one to enumerate all the relations between areas, lines and points and is known as the "dimension entended method"(DEM). A very large number of possible relationships may be defined in this way and a way termed as the "calculus based method"(CBM) to generate all these from a set of five polymorphic binary relations between a pair of spatial entities x and y : disjoint, touch, in, overlap, cross has been proposed [25]. A complex relation between x and y may then be formed by conjoining atomic propositions formed by using one of the five relations above, whose arguments may be either x or y or a boundary or endpoint operator applied to x or y. For the most expressive calculus (either the CBM or the combination of the 9-intersection and the DEM) there are 9 JEPD area/area relations, 31 line/area relations, 3 point/area relations, 33 line/line relations, 3 point/line relations and 2 point/point relations giving a total of 81 JEPD relations [25].

4.3. Modes of Overlap

There are a variety of ways in which two regions can partially overlap each other [87]. In most previous work (an exception is [42]), partial overlap has always been taken to be a single relation (usually denoted as $\mathsf{PO}(x,y)$), just as connection itself is usually taken to be a single relation. Whilst realizing that there are potentially infinitely many varieties of partial overlap relation, Galton parameterized these using a matrix notation:

$$\begin{pmatrix} x & a \\ b & o \end{pmatrix}$$

where x, a, b and o are the numbers of connected components of $x \cap y$, $x \setminus y$, $y \setminus x$, $compl(x \cup y)$. He investigates all matrices with numbers no greater than two; of the 54 theoretical possibilities, just 23 are physically realisable.

4.3.1. Mereology and Topology

Varzi [175, 174] presents a systematic account of the subtle relations between mereology and topology. He notes that whilst mereology is not sufficient by itself, there are theories in literature which have proposed integrating topology and mereology. There are three main strategies of integrating the two :

- Generalize mereology by adding a topological primitive. Borgo et. al. [16] add the topological primitive $\mathsf{SC}(x)$, i.e., x is a self connected (one-piece) spatial entity to the mereological part relation. Alternatively a single primitive can be used as in [175]: "x and y are connected parts of z". The main advantage of separate theories of mereology and topology is that it allows collocation without sharing parts which is not possible in the second two approaches below.
- Topology is primal and mereology is a sub theory. For example in the topological theories based on $\mathsf{C}(x,y)$, such as those mentioned above, one defines $\mathsf{P}(x,y)$ from $\mathsf{C}(x,y)$. This has the elegance of being a single unified theory, but colocation implies sharing of parts. These theories are normally boundaryless (i.e. without lower dimensional spatial entities) but this is not absolutely necessary [152, 100].
- The final approach is that taken by [67], i.e. topology is introduced as a specialised domain specific sub theory of mereology. An additional primitive needs to be introduced. The idea is to use restricted quantification by introducing a sortal predicate $\mathsf{Region}(x)$. $\mathsf{C}(x,y)$ can then be defined thus: $\mathsf{C}(x,y) \equiv_{def} \mathsf{O}(x,y) \wedge \mathsf{Region}(x) \wedge \mathsf{Region}(y)$.

4.4. Between Topology and Fully Metric Spatial Representation

Mereology and Topology can be seen as perhaps the most abstract and most qualitative spatial representations. However, although potentially useful there are many situations where mereotopological information alone is insufficient. The following sections explore the different ways in which other qualitative information may be represented.

4.4.1. Orientation

Orientation relations describe where objects are placed relative to one another, and can be defined in terms of three basic concepts: the primary object, the reference object and the frame of reference. Thus, unlike the topological relations on spatial entities described in the preceding sections, orientation is not a binary relation i.e., if we want to specify the orientation of a *primary object*(PO) with respect to a *reference object*(RO), then we need to have some kind of a *frame of reference*(FofR). This characterization manifests itself in the display of qualitative orientation calculi to be found in the literature: certain calculi have an explicit triadic relation while others presuppose an extrinsic frame of reference [78, 113].

Of those with explicit triadic relations it is especially worth mentioning the work of Schlieder [160], following earlier work by [96], who develops a calculus based on function which maps triples of points to one of three qualitative values, $+, 0$ or $-$, denoting anticlockwise, collinear and clockwise orientations respectively. This can be used for reasoning about visible locations in qualitative navigation tasks or for shape descriptions [162]. Schlieder develops a calculus [161] for reasoning about the relative orientation of pairs of line segments.

A triadic orientation calculus, based on a relation CYCORD(x,y,z) which is true (in 2D) when x, y, z are in a clockwise orientation, shows how a number of qualitative calculi can be translated into the CYCORD system [158], whose reasoning system (implemented as a constraint logic program) can then be exploited. The disadvantage of the CYCORD theory is that reasoning in it is NP complete. A refinement of the theory, leading to an algebra of ternary relations for cyclic ordering of 2D orientations contains 24 atomic relations, hence 2^{24} general relations, of which CYCORD relation is one [117]. However, the propogation algorithm is polynomial[8], and complete for a subclass including all atomic relations.

4.4.2. Distance and Size

Spatial representation of distance can be divided into two main groups: those which measure on some "absolute" scale, and those which provide some kind of relative measurement. Of course, since traditional Qualitative Reasoning [180] is primarily concerned with dealing with linear quantity spaces, the qualitative algebras and the transitivity of such quantity spaces mentioned earlier can be used as a distance or size measuring representation.

Also of interest in this context are the order of magnitude calculi [134, 144] developed in the QR community. Most of these traditional QR formalisms are of the "absolute" kind of representations[9], as in the delta calculus of [186] - which introduces a triadic relation: $x(>, d)y$ to note that x is larger/bigger than y by an amount d; terms such as $x(>, y)y$ mean that x is more than twice as big as y.

Of the "relative" representations specifically developed within the qualitative spatial reasoning community, perhaps the earliest is the triadic $\mathsf{CanConnect}(x, y, z)$ primitive [51] – which is true if body x can connect y and z by simple translation (i.e., without scaling, rotation or shape change). From this primitive it is easy to define notions such as equidistance, nearer than and farther than. This primitive allows a simple size metric on regions to be defined: one region is larger than another if it can connect regions that the other cannot. Another method of determining the relative size of two objects relies on being able to translate regions (assumed to be shape and size invariant) and then exploit topological relationships – if a translation is possible so that one region becomes a proper part of another, then it must be smaller [136].

Distance is closely related to the notion of orientation: e.g. distances cannot usually be summed unless they are in the same direction, and the distance between a point and a region may vary depending on the orientation. Thus it is perhaps not surprising that there have been a number of calculi which are based on a primitive which combines distance and orientation information. One straightforward idea [78] is to combine directions as represented by segments of the compass with a simple distance metric (*far, close*). A slightly more sophisticated idea is to introduce a primitive which defines the position of a third point with respect to a directed line segment between two other points [187]. Another approach that combines knowledge about distances and positions in a qualitative way - through a combination of the Delta-calculus [186] and orientation is presented in [185].

Liu [129] explicitly defines the semantics of qualitative distance and qualitative orientation angles and formulates a representation of qualitative trigonometry. He defines *composition tables* for the calculus to combine both types of information and suggests computing a quantitative visualization by simulated annealing.

Of particular interest is the framework for representing distance [114] which has been ex-

[8] Complexity results for some other QSR calculi are presented in Section 5.2

[9] Actually it is straightforward to specify relative measurements given an "absolute" calculus: to say that $x > y$, one may simply write $x - y = +$.

tended to include orientation [31][10]. A distance system is composed of an ordered sequence of *distance relations* and a set of *structure relations* which give additional information about how the distance relations relate to each other. Each distance has an *acceptance area*; the distance between successive acceptance areas defines sequence of intervals: $\delta_1, \delta_2, \ldots$ The structure relations define relationships between these δ_i. Typical structure relations might specify a monotonicity property (the δ_i are increasing), or that each δ_i is greater than the sum of all the preceding δ_i. The structure relationships can also be used to specify order of magnitude relationships, e.g. that $\delta_i + \delta_j \sim \delta_i$ for $j < i$. The structure relationships are important in refining the *composition tables*[11]. In a *homogeneous* distance system all distance relations have the same structure relations; however this need not be the case in a *heterogeneous* distance system. The proposed system also allows for the fact that the context may affect the distance relationships: this is handled by having different frames of reference, each with its own distance system and with inferences in different frames of reference being composed using *articulation rules* (cf. [116]).

One obvious effect of moving from one scale, or context to another, is that qualitative distance terms such as "close" will vary greatly; more subtly, distances can behave in various "non-mathematical" ways in some contexts or spaces: e.g. distances may not be symmetrical[12]. Another "mathematical aberration" is that in some domains the shortest distance between two points may not be a straight line (e.g. because a lake or a building might be in the way).

4.4.3. Shape

Shape is perhaps one of the most important characteristics of an object, and particularly difficult to describe qualitatively. In a purely topological theory very limited statements can be made about the shape of a region: whether it has holes, or interior voids, or whether it is one piece or not. It has been observed [83] that one can (weakly) constrain the shape of rigid objects by topological constraints using RCC-8 relations.

However, if an application demands finer grained distinctions, then some kind of semi-metric information has to be introduced[13]. For an explicit qualitative shape description one needs to go beyond topology, introducing some kind of shape primitives whilst still retaining a qualitative representation. Of course, as [30] note: the mathematical community have developed many different geometries which are less expressive than Euclidean geometry, for example projective and affine geometries, but have not necessarily developed efficient computational reasoning techniques for them[14].

Of the qualitative approaches to shape description, approaches which work by describing the boundary of an object include those that classify the sequence of different types of bound-

[10] Whereas [31] combines qualitative orientation and absolute distance knowledge, [118] combines qualitative orientation [117] and relative distance information. Another example of a combined distance and position calculus is [68].

[11] Section 5 introduces composition tables

[12] E.g. because distances are sometimes measured by time taken to travel, and an uphill journey may take longer than a return downhill journey [114].

[13] Of course, orientation and distance primitives as discussed above already add something to pure topology, but as already mentioned these are largely point based and thus not directly applicable to describing shape of a region.

[14] Though see [5, 6].

ary segments [157] or by describing the sequence of different types of curvature extrema [126] along its contour. Another technique is described by [120] who uses a slope projection approach to describe polygonal shape: for each corner one describes whether it is convex/concave, obtuse/right-angled/acute together with a qualitative representation of the direction of the corner (chosen from a set of nine possible values), as well as the multi-resolution approach of [22]. Alternatively one might construct a complex shaped region out of simpler ones along the lines of constructive solid geometry, but starting from a more qualitative set of primitives [156].

A dichotomy can be drawn between representations which primarily describe the boundary of an object compared to those which represent its interior. Arguably the latter techniques are preferable since shape is inherently not a dimensional concept [17]. A version of the medial axis transform, defining grain of a region within a shape, leads to a modified version of the traditional skeleton, called the angular skeleton [56]. Shape description using volume components is exemplified by the work of [132, 14].

The shape abstraction primitives such as the bounding box or the convex hull have been considered briefly within the 9-intersection model [29] whilst the latter technique has been investigated extensively within the RCC calculus [32, 42, 50]. RCC theory has shown that many interesting predicates can be defined once one take the notion of a convex hull of a region (or equivalently, a predicate to test convexity) and combines it with a topological representation. By computing the topological relationships between the shape itself and the different components of the difference between the convex hull and the shape, one can distinguish many different kinds of concave shapes [32]. A refinement to this technique exploits the idea of recursive shape description [165] to describe any non convex components of the difference between the convex hull and the shape. The convex hull is clearly a powerful primitive and in fact it has recently been shown [50] that this system essentially is equivalent to an affine geometry: any two compact planar shapes not related by an affine transformation can be distinguished by a constraint language of just $\mathsf{EC}(x, y)$, $\mathsf{PP}(x, y)$ and $\mathsf{conv}(x)$.

Before leaving the topic of shape description we should point out the work of [30] on describing shape via properties such as compactness and elongation by using the minimum bounding rectangle of the shape and the order of magnitude calculus of [134]: elongation is computed via the ratio of the sides of the minimum bounding rectangle whilst compactness by comparing the area of the shape and its minimum bounding rectangle. The notion of a voronoi hull has also been used [55].

4.5. Vagueness, Uncertainty and Granularity

Vagueness, Uncertainty and Granularity are features of information which permeate almost every domain of knowledge representation. Uncertainty and vagueness are endemic in many applications, for example because of indeterminate region boundaries. Granularity follows from the notion that for some purposes only very general descriptions are required[15], whereas in other cases one would like as much precision as possible.

Vagueness of spatial concepts can be distinguished from that associated with spatially situated objects and the regions they occupy. An adequate treatment of vagueness in spatial

[15] A theory of abstraction [91] – i.e, going from detailed to general information – has received particular attention from the AI community.

information will need to account for vague regions as well as vague relationships [40]. Although there has been some philosophical debate concerning whether vague objects can exist [70], formal theories dealing with vagueness of extent are not well-established.

As for uncertainty, it is always possible to glue on some standard numerical technique for reasoning about uncertainty (e.g. [82]), but there has also been some research on extending existing qualitative spatial reasoning techniques to explicitly represent and reason about uncertain information. Proposed extensions of the RCC calculus [39] and the 9-intersection [28] take a very similar approach. The former approach which is continued in a series of papers [37, 38, 40] postulates the existence of non crisp regions in addition to crisp regions and then adds another binary relation to RCC - x is crisper than region y. A variety of relations are then defined in terms of this primitive and this extended theory is then related to what has become known as the "egg-yolk" calculus, which originated in [125], to represent and reason about indefiniteness of a region's boundaries. Essentially, it models regions with indeterminate boundaries as a pair of regions. As shown in Figure 4, the yolk represents a minimum extension of the indefinite region, whilst the egg represents its maximum extension. It turns out that if one generalizes RCC-8 in this way [40] there are 252 JEPD relations between non crisp regions which can be naturally clustered into 40 sets.

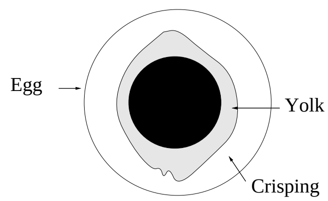

Figure 4. An egg-yolk structure for regions with indeterminate boundaries.

The extension of the 9-intersection [28] looks very similar to the egg-yolk calculus but does not consider such fine granularity of relations; it postulates 44 JEPD relations, also clustered into groups (18 in their case) but using a more ad hoc technique to achieve this. An interesting extension to this work [29] shows that this calculus of regions with broad boundaries can be used to reason not just about regions with indeterminate boundaries but also can be specialised to cover a number of other kinds of regions including convex hulls of regions, minimum bounding rectangles, buffer zones and rasters[16].

Another notion of indefiniteness relates to locations. Bittner [15] deals with the notion of exact, part and rough location for spatial objects. The exact location is the region of space taken up by the object. The notion of part location (as introduced by [19]) relates parts of a spatial object to parts of spatial regions. The rough location of a spatial object is characterized by the part location of spatial objects with respect to a set of regions of space that form regional partitions. Consequently, the notion of rough location links parts of spatial objects to parts of partition regions.

Bittner [15] argues that the observations and measurements of location in physical reality yield knowledge about rough location: a vaguely defined object o is located within a regional

[16] This last specialization generalises the application of the n-intersection model to rasters previously undertaken [66].

partition consisting of the three concentric regions: 'core', 'wide boundary' and 'exterior'. In this context, the notion of rough location within a partition consisting of the three concentric regions coincides with the notion of vague regions introduced by [39]. A calculus for representing and reasoning about the location of rigid objects which may move within some region is presented in [46] which presents an axiomatisation for congruence, defines the notion of a mobile part, describes a subset of morpho-mereological relations suitable for representing spatial locations, and analyzes the computational complexity of this set. Further work based on congruence is [11, 12].

It is worth noting the similarity of these ideas to rough sets [53], though the exact relationship has yet to be explored. Other approaches to spatial uncertainty are to work with an indistinguishability relation which is not transitive and thus fails to generate equivalence classes [173, 121] and the development of nonmonotonic spatial logics [163, 2].

5. Qualitative Spatial Reasoning

Much of the work in QSR has concentrated on representational aspects. Nevertheless, various computational paradigms are also being investigated including constraint based reasoning (e.g. [113]). The most prevalent form of qualitative reasoning is based on the composition table[17].

A compositional inference is a deduction, from two relational facts of the form $R_1(a,b)$ and $R_2(b,c)$, of a relational fact of the form $R_3(a,c)$, involving only a and c. The validity of compositional inferences does not depend in many cases on the constants involved but only on the logical properties of the relations. In such a case the composition of pairs of relations can be maintained for table look up as and when required. This technique is of particular significance when we are dealing with relational information involving a fixed set of relations. Given a set of n JEPD relations, one can store in a n x n composition table the relationships between x and z for a pair of relations $R_1(x,y)$ and $R_2(y,z)$. In general, each entry will be a disjunction because of the qualitative nature of the calculus.

For a representation using a limited set of binary relations, the simplicity of the compositional inference makes it an attractive means of effective reasoning. Since their introduction, composition tables have received considerable attention from researchers, e.g. [178, 60, 79, 149, 158, 36, 161]. Most of the calculi mentioned in this paper have had composition tables constructed for them, though it has at times posed something of a challenge [149].

A reformulation of the RCC first order logic into a zero order representation[18] [7] in which intuitionistic propositional formulae represent constraints on possible situations was able to generate the composition table as shown in Table 1 automatically[19].

Composition tables do not necessarily subsume all forms of desired reasoning. An interesting question than arises: exactly when is a composition table reasoning a sufficient inference

[17]Originating in Allen's analysis of temporal relations and called the 'transitivity table' [1], now more appropriately renamed 'composition table' since more than one relation is involved and it is the composition of the relations that is being represented rather than the transitivity of individual relations.

[18]This reformulation is interesting in that it becomes a true spatial logic, rather than a theory of space: the "logical symbols" have spatial interpretations, e.g. implication is interpreted as parthood and disjunction as the sum of two regions.

[19]Another approach for automatic generation of composition tables uses a zero order modal logic [9].

R2(b,c) / R1(a,b)	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ
DC	no.info	DR,PO,PP	DR,PO,PP	DR,PO,PP	DR,PO,PP	DC	DC	DC
EC	DR,PO,PPi	DR,PO TPP,TPi	DR,PO,PP	EC,PO,PP	PO,PP	DR	DC	EC
PO	DR,PO,PPi	DR,PO,PPi	no.info	PO,PP	PO,PP	DR,PO,PPi	DR,PO PPi	PO
TPP	DC	DR	DR,PO,PP	PP	NTPP	DR,PO TPP,TPi	DR,PO PPi	TPP
NTPP	DC	DC	DR,PO,PP	NTPP	NTPP	DR,PO,PP	no.info	NTPP
TPPi	DR,PO,PPi	EC,PO,PPi	PO,PPi	PO,TPP,TPi	PO,PP	PPi	NTPPi	TPPi
NTPPi	DR,PO,PPi	PO,PPi	PO,PPi	PO,PPi	O	NTPPi	NTPPi	NTPPi
EQ	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ

Table 1. Composition table for the RCC-8 relations

mechanism (i.e for which theories is it complete)? This has been taken up in [54] and in [10] it has been suggested that the formalism of relational algebra is particularly suited to representing compositional reasoning and deserves further study and wider application.

For cases when composition based reasoning is not sufficient, then other more general constraint based reasoning may be sufficient [113, 106] or as in [69] where QSR is seen as a constraint satisfaction problem[20]. More generally, one may resort to theorem proving, or preferably, some kind of specialised theorem proving system, [7, 158, 108] for example.

5.1. Reasoning about Spatial Change

So far we have been concerned purely with static spatial calculi. However it is important to develop calculi which combine space and time in an integrated fashion. Topological changes in 'single' spatial entity include: change in dimension (this is usually 'caused' by an abstraction or granularity shift rather than an 'actual' spatial change[21]); change in number of topological components (e.g. breaking a cup, fusing blobs of mercury); change in the number of tunnels (e.g. drilling through a block of wood); change in the number of interior cavities (e.g. putting a lid on a container).

In many domains we assume that change is continuous[22], as is the case in traditional qualitative reasoning, and thus there is a requirement to build into the spatial calculus which changes in value will respect the underlying continuous nature of change, and this requirement is of course common to all the different kinds of spatial change we have mentioned above. It is thus important to know which qualitative values or relations are neighbours in the sense that if a value or predicate holds at one time, then there is some continuous change possible such that the next value or predicate to hold will be a neighbour. *Continuity networks* defining such neighbours are often called *conceptual neighbourhoods* in the literature following the use

[20] As a consequence of this, the treatment of several aspects of space (such as orientation following the Freksa and Zimmermann's approach [80], cardinal directions following Frank's approach [78] and qualitative named distances [80] could be integrated into the same spatial model.

[21] E.g. we may view a road as being a 1D line on a map, a 2D entity when we consider whether it is wide enough for an outsize load, and a 3D entity as we consider the range of mountains it passes over, or the potholes and a particularly delicate cargo.

[22] Sometimes change is discontinuous, e.g. when political fiat moves the boundaries of geopolitical entities in a discontinuous manner.

of the term [79] to describe the structure of Allen's 13 JEPD relations [1] according to their conceptual closeness[23](e.g. *meets* is a neighbour of both *overlaps* and *before*). As illustrated in Figure 1 above, most of the qualitative spatial calculi reported in this paper have had conceptual neighbourhoods constructed for them[24].

5.1.1. Qualitative Simulation

Perhaps the most common form of computation in the traditional QR literature is qualitative simulation; using conceptual neighbourhood diagrams it is easy to build a qualitative spatial simulator [47]. Such a simulator takes a set of ground atomic statements describing an initial state[25] and constructs a tree of future possible states – the branching of the tree results from ambiguity of the qualitative calculus. Of course, continuity alone does not provide sufficient constraints to restrict the generation of next possible states to a reasonable set in general – domain specific constraints are required in addition.

A desirable extension by analogy with earlier QR work would be to incorporate a proper theory of spatial processes couched in a language of QSR. Some work in this direction is: a field based theory of spatial processes such as heat flow [130]; an analysis of which traversals of a topological conceptual neighbourhood diagram correspond to processes such as expansion of a region, rotation of a region etc [62]; how the processes of protrusion and resistance cause changes in a boundary based shape description language – given two shapes one can infer sequences of processes which could cause one to change into the other [126]. Also worthy of note is the qualitative spatial simulation work of [145] based on the QSIM system [180].

5.1.2. Qualitative Motion

Inspite of a large amount of work in mereo-topological theories as a basis for common-sense reasoning, very little work has been done on motion in a qualitative framework. Representation of motion as in [146, 112] is in a Newtonian/Cartesian framework. Some work [75, 49] insist more on the concept of dynamic processes; elsewhere [163, 109] there is research on default reasoning in metric spaces that are not really clearly characterized.

Motion is nevertheless a key notion in our understanding of spatial relations. RCC relations form "conceptual-neighbourhoods" via potential motion, but continuity remains an implicitly assumed notion. One problem is that the *conceptual neighbourhood* is usually built manually for each new calculus – potentially an arduous and error prone operation if there are many relations. Techniques to arrive at these automatically would be very useful. An analysis of the structure of *conceptual neighbourhoods* reported by [127] goes some way towards this goal. A more fundamental approach which exploits the continuity of the underlying semantic spaces [86]

---

[23] Note that one can lift this notion of closeness from individual relations to entire scenes via the set of relations between the common objects and thus gain some measure of their conceptual similarity as suggested by [18].

[24] A closely related notion is that of "closest topological distance" [62] – two predicates are neighbours if their respective n-intersection matrices differ by fewer entries than any other predicates; however the resulting neighbourhood graph is not identical to the true conceptual neighbourhood or continuity graph - some links are missing.

[25] The construction of an envisioner [180] rather than a simulator would also be possible of course. See also the transition calculus approach of [94].

not only allows the construction of a conceptual neighbourhood for a class of relations from a semantics, but also infers which relations dominate other relations: R_1 dominates R_2 if R_2 can hold over an interval followed/preceded by R_1 instantaneously. E.g. in RCC-8 TPP dominates NTPP and PO, while EQ dominates all its neighbouring relations. Dominance is analogous to the equality change law to be found in traditional QR [180] and allows a stricter temporal order to be imposed on events occuring in a qualitative simulation. Motivated by the desire to exploit decidable modal logics for spatio-temporal qualitative reasoning, a series of rather expressive such calculi have been proposed [182] in which it is possible easily to represent restrictions on continuous motion.

An approach to automatically inferring continuity networks has been proposed by Muller [138, 139, 137]. Taking up the idea of spatio-temporal histories [111], he enriches a theory intended for spatial entities to achieve a formal theory for reasoning about motion and presents a mereo-topological model in which the primitive entities are spatio-temporal regions, on which spatio-temporal and temporal relations are defined. The expressive power of the theory allows for the definition of complex motion classes such as those expressed by motion verbs in natural language. Properties usually assumed as desirable parts of any space-time theory of motion are recovered from their model, thus giving a sound theoretical basis for a natural, qualitative representation of motion. An alternative approach is [12] which explores the expressive power of *Region-Based Qualitative Geometry* [11] to the problem of representing and reasoning about the motion of rigid bodies within a confining environment[26].

5.2. Theoretical Results in QSR

Even though much work has been done in generating spatial representational calculi, there remain a number of theoretical questions. Not all calculi has been given a formal semantics by their inventors. Even for those that has been given, there is the question of whether it is the best or simplest semantics. Further, given a semantics one can ask whether the task of showing a set of formulae is consistent or whether one set entails another is decidable, and if so, what is the complexity of the decision procedure. One can ask if the theory is complete, either in the weak sense of every true formula being provable, or in the stronger sense of whether every formula is made either true or false in the theory. Any complete, recursively axiomatizable theory is decidable. Finally, there is the property of being categorical, i.e. whether all models are isomorphic? Since theories may have models of various cardinalities, and models of different cardinalities cannot by definition be isomorphic, a more interesting property is $\aleph_0$ categoricity i.e. whether all countable models are isomorphic, since these are perhaps the most useful models from the user's viewpoint.

A question arises as to whether there is something special about region based theories from the ontological viewpoint? The answer is in the negative, at least for 2D mereotopology [142]: they show, under certain assumptions, that the standard 2D point based interpretation is the

[26] The approach is based on the morpho-topological approach in [16] but has some significant advantages. Firstly [11] assume only *parthood* and a morphological primitive (which may be either the sphere predicate or the congruence relation), whereas [16] employ an additional topological primitive ('simple region'). Topological concepts are nevertheless definable in [11] and they also prove categoricity by a more direct encoding of Tarski's geometry axioms into the language.

simplest model (prime model); the only alternative models involve regions with infinitely many pieces. But it may be argued that it is still useful to have region based theories even if they are always interpretable point set theoretically.

A fundamental result on decidability which has widespread applicability in qualitative spatial theories is that of [105] which shows that although Boolean algebra is decidable; adding either a closure operation or an external connection relation results in an undecidable system since one can then encode arbitrary statements of arithmetic. This implies that Clarke's calculus and all related calculi such as the first order theory of RCC, and the calculi of [3] and [16] are all undecidable.

The question then becomes whether there are any decidable subsystems[27]? The constraint language of RCC-8 has been shown to be decidable [7] – this was achieved by encoding each RCC-8 relation as a set of formulae in intuitionistic proportional calculus which is a decidable calculus. This language was subsequently shown to be tractable [140] – in fact the satisfaction problem is solvable in polylogarithmic time since it is in the complexity class NC. However the constraint language of $2^{\mathrm{RCC\text{-}8}}$ (i.e where constraints may be arbitrary disjunctions of RCC-8 relations) is not tractable.

A maximal tractable subset (containing 148 relations) of the constraint language of $2^{\mathrm{RCC\text{-}8}}$ has been identified and furthermore have shown that path consistency is sufficient for deciding consistency in this case [155]. If an appropriate size constraint is introduced between two regions then all reasoning in $2^{\mathrm{RCC\text{-}8}}$ effectively becomes polynomial [89].

More recently, a complete classification of the tractability of RCC-8 has been made [153]. It turns out that there are two further maximal tractable subsets (containing 158 and 160 relations respectively) and these three subsets are the only such sets for RCC-8 that contain all base relations. Further work on the complexity of RCC includes [45].

As far as the complexity of non topological theories is concerned, [117] presents and analyses an orientation calculus and determines polynomial subsets (including all the base relations), whilst determining satisfiability in the general algebra is NP complete. Similarly [128] shows that whilst the general consistency problem in the algebra of cardinal directions is NP complete, consistency for the preconvex relations is polynomial and this set is a maximal tractable subset. A complexity analysis of an morphological calculus based on congruence is [119].

Also of interest is the analysis of [104] which considers an RCC-8 like calculus and two simpler calculi and determines which of a number of different problem instances of relational consistency and planar realizability are tractable and which are not – the latter is the harder problem. It has also been shown that the constraint language of $\mathsf{EC}(x,y)$, $\mathsf{PP}(x,y)$ and $\mathsf{conv}(x)$ is intractable (it is at least as hard as determining whether a set of algebraic constraints over the reals is consistent) [50].

Clarke's system [24] has been given a semantics (regular sets of Euclidean space are models) and has shown to be complete in the weak sense [13]. Unfortunately it turns out that contrary to Clarke's intention, only mereological relations are expressible! The theory in fact characterises a complete atomless Boolean algebra. The system of [3] which corrects the problems in Clarke as mentioned above, is given a semantics and shown to be complete by the authors but their

[27]Rather in the same manner as the description logic community have sought to find the lines dividing decidability from undecidability and tractability from intractability [21].

inclusion of the notion of 'weak connection' forces a non standard model since models must be non dense[28].

A completeness result (in the strong sense) has been derived for a complete 2D topological theory whose elements are 2D finite (polygonal) regions and whose primitives are: the null and universal regions, the Boolean functions $(+, *, -)$ and a predicate to test for a region being one piece [142]; the theory is first order but requires an infinitary rule of inference. This is not surprising in view of the undecidability of first order topology mentioned above [105]. The infinitary rule of inference guarantees the existence of models in which every region is sum of infinitely many connected regions. The resulting theory is complete but not decidable.

Notwithstanding the attempt [8] to derive a complete first order topological theory, it is now clear that no first order finite axiomatisation of topology can be complete or categorical because it is not decidable.

6. Final Comments

An issue that has not been much addressed yet in the QSR literature is the issue of cognitive validity. Claims are often made that qualitative reasoning is akin to human reasoning, but with little or no empirical justification. One exception to this is the study made of a calculus for representing topological relations between regions and lines [131]. Another study is [123] that have investigated the preferred Allen relation (interpreted as a 1D spatial relation) in the case that the composition table entry is a disjunction. Perhaps the fact that humans seem to have a preferred model explains why they are able to reason efficiently in the presence of the kind of ambiguity engendered by qualitative representations. In [122] they extend their evaluation to RCC-theory and Egenhofer's approach to topological relations.

In this paper we have surveyed many of the key ideas and results in the QSR literature, but space has certainly not allowed an exhaustive survey. As in so many other fields of knowledge representation it is unlikely that a single universal spatial representation language will emerge – rather, the best we can hope for is that the field will develop a library of representational and reasoning devices and some criteria for their most successful application. Moreover, as in the case of non spatial qualitative reasoning, quantitative knowledge and reasoning must not be ignored – qualitative and quantitative reasoning are complementary techniques and research is needed to ensure they can be integrated – for example by developing reliable ways of translating between the two kinds of formalisms [29]. Equally, interfacing symbolic QSR to the techniques being developed by the diagrammatic reasoning community [92] is an interesting and important challenge.

---

[28] This enforced abondonment of R^n as a model leads one to question whether it is indeed a good idea to try to model the proposed distinction between strong and weak connection topologically in a purely spatial theory, rather than in an applied theory of physical objects and material substances together with the regions they occupy. They do propose an extension to their theory in which they allow the spatial granularity to be varied; as finer and finer granularities are considered, so fewer instances of weak connection are true and in the limit the theory tends to the classical topological model.

[29] Some existing research on this problem includes [77, 73, 169].

7. Acknowledgements

The first author gratefully acknowledges the financial support of several EPSRC grants (most recently GR/M56807) and the EU support of the SPACENET network. The SPACENET workshops provided an excellent forum for discussion of many of the ideas in this paper and contributions from the SPACENET community and colleagues at Leeds, in particular Brandon Bennett, are very gratefully acknowledged. The second author would like to acknowledge the financial assistance under reference INCS-1999-177 received from Commonwealth Scholarship Commission, United Kingdom.

References

[1] Allen, J.F.: "Maintaining Knowledge about Temporal Intervals", *Communications of the ACM*, **26**(11), 1983, pages 823–843.

[2] Asher, N. and Lang, J.: "When nonmonotonicity comes from distance", B. Nebel and L. Dreschler-Fischer (eds.) *KI-94: Advances in Artificial Intelligence*, Springer-Verlag, 1994, pages 308–318.

[3] Asher, N. and Vieu, L.: "Toward a geometry of common sense: A semantics and a complete axiomatization of mereotopology", *Proc. 14th Intl. Joint Conf. on Artificial Intelligence (IJCAI-95)*, Montreal, 1995, pages 846–852.

[4] Aurnague, M. and Vieu, L.: "A three-level approach to the semantics of space", C. Zelinsky-Wibbelt (ed.) *The semantics of prepositions - from mental processing to natural language processing*, Mouton de Gruyter, Berlin, 1993.

[5] Balbiani, B., Dugat, V., Farninas del Cerro, L. and Lopez, A.: *Eléments de géométrie mécanique*, Edition Hermes, 1994.

[6] Balbiani, P., Fariñas del Cerro, L., Tinchev, T. and Vakarelov, D.: "Modal Logics for Incidence Geometries", *Journal of Logic and Computations*, **7**(1), 1997, pages 59–78.

[7] Bennett, B.: "Spatial Reasoning with Propositional Logics", J. Doyle, E. Sandewall and P. Torasso (eds.) *Principles of Knowledge Representation and Reasoning : Proc. 4th Intl. Conf. (KR'94)*, San Francisco, Morgan Kaufmann, 1994.

[8] Bennett, B.: "Carving Up Space: steps towards construction of an absolutely complete theory of spatial regions", J.J. Alfres, L.M. Pereira and E. Orlowska (eds.) *Proc. JELIA'96*, 1996, pages 337–353.

[9] Bennett, B.: "Modal Logics for Qualitative Spatial Reasoning", *Bulletin of the Interest Group in Pure and Applied Logic (IGPL)*, 4(1), 1996, pages 23–45. `ftp://ftp.mpi-sb.mpg.de/pub/igpl/Journal/V4-1/index.html`

[10] Bennett, B., Isli, A. and Cohn, A. G. : "When does a composition table provide a complete and tractable proof procedure for a relational constraint language?", *Proc. IJCAI-97 Workshop on Spatial and Temporal Reasoning*, Nagoya, Japan, 1997.

[11] Bennett, B., Cohn, A. G., Torrini, P. and Hazarika, S. M.: "A Foundation for Region-Based Qualitative Geometry", *Proc. 14th European Conf. on AI (ECAI-2000)*, Berlin, Germany, 2000.

[12] Bennett, B., Cohn, A. G., Torrini, P. and Hazarika, S. M.: "Describing Rigid Body Motions in a Qualitative Theory of Spatial Regions", *Proc. 17th American Conf. on AI (AAAI-2000)*, Austin, Texas, 2000.

[13] Biacino, L. and Gerla, G.: "Connection Structures", *Notre Dame Journal of Formal Logic*, **32**(2), 1991, pages 242-247.

[14] Biederman, I.: "Recognition by components: A theory of human image understanding", *Psychological Review*, **94**(2), 1987, pages 115–147.

[15] Bittner, T.: "On Ontology and Epistemology of Rough Location", C. Freksa and D. M. Mark (eds.) *Spatial Information Theory - Cognitive and Computational Foundations of Geographic Information Science: Proc. COSIT'99*, LNCS No. 1661, Springer, 1999, pages 433–448.

[16] Borgo, S., Guarino, N. and Masolo, C.: "A pointless theory of space based on strong connection and congruence", L. Carlucci Aiello and J. Doyle (eds.) *Principles of Knowledge Representation and Reasoning: Proc. 5th Intl. Conf. (KR'96)*, Morgan Kaufman, 1996, pages 220-229.

[17] Brady, J. M.: "Criteria for representations of shape", *Human and Machine Vision*, 1993.

[18] Bruns, H. T. and Egenhofer, M.: "Similarity of spatial scenes", *Proc. 7th Intl. Symposium on Spatial Data Handling, (SDH'96)*, Francis Taylo, 1996, page 173-184.

[19] Casati, R. and Varzi, A.: "The structure of spatial localization", *Philosophical Studies*, **2**(82), 1991, pages 205–239.

[20] Casati, R. and Varzi, A.: *Holes and Other Superficialities*, MIT Press, Cambridge, MA, 1994.

[21] D. Calvanese, G. De Giacomo, M. Lenzerini and D. Nardi: "Reasoning in Expressive Description Logics", A. Robinson and A. Voronkov (eds.) *Handbook of Automated Reasoning*, Elsevier Science Publishers (North-Holland), Amsterdam, 1999.

[22] Cinque, L. and Lombardi, L.: "Shape description and recognition by a multiresolution approach", *Image and Vision Computing*, **13**(8), 1995, pages 559–607.

[23] Clarke, B. L.: "A calculus of individuals based on 'connection"', *Notre Dame Journal of Formal Logic*, **22**(3), 1981, pages 204–218.

[24] Clarke, B. L.: "Individuals and points", *Notre Dame Journal of Formal Logic*, **26**(1), 1985, pages 61–75.

[25] Clementini, E., Di Felice, P. and Oosterom, P.: "A small set of formal topological relationships suitable for end user interatction", D. Abel and B. C. Ooi (eds.) *Advances in Spatial Databases: Proc. 3rd Intl. Symposium on Spatial Databases (SSD'93)*, LNCS No. 692, Springer-Verlag, 1993, pages 277–295.

[26] Clementini, E., Sharma, J. and Egenhofer, M. J.: "Modeling topological spatial relations: strategies for query processing", *Computers and Graphics*, **18**(6), 1994, pages 815–822.

[27] Clementini, E. and Di Felice, P.: "A Comparison of Methods for Representing Topological Relationships", *Information Sciences*, **3**, 1995, pages 149–178.

[28] Clementini, E. and Di Felice, P.: "An algebraic model for spatial objects with undetermined boundaries", P. Burrough and A. M. Frank (eds.) *Proc. GISDATA Specialist Meeting on Geographical Entities with Undetermined Boundaries*, Francis Taylor, 1996.

[29] Clementini, E. and Di Felice, P. : "Approximate Topological Relations", *International Journal of Approximate Reasoning*, 1997.

[30] Clementini, E. and Di Felice, P.: "A global framework for qualitative shape description", *GeoInformatica*, **1**(1), 1997.

[31] Clementini, E., Di Felice, P. and Hernández, D.: "Qualitative Representation of Positional Information", *Artificial Intelligence*, **95**(2), 1997, pages 317–356.

[32] Cohn, A. G.: "A hierarchical representation of qualitative space based on connection and connectivity", A. Frank and W. Kuhn (eds.) *Spatial Information Theory: A theoretical basis for GIS, Proc. COSIT'95*, LNCS No. 988, Springer-Verlag, 1995, pages 311-326.

[33] Cohn, A. G.: "Qualitative spatial representation and reasoning techniques", G. Brewka, C. Habel and B. Nebel (eds.) *KI-97: Advances in Artificial Intelligence*, LNAI No. 1303, Springer, 1997, pages 1–30.

[34] Cohn, A. G.: "Qualitative Spatial Representations", *Proc. IJCAI-99 Workshop on Adaptive Spatial Representations of Dynamic Environment*, 1999, pages 33–52.

[35] Cohn, A. G., Bennett, B., Gooday, J. and Gotts, N. M.: "Representing and reasoning with qualitative spatial relations about regions", O. Stock (ed.) *Spatial and Temporal reasoning*, Kluwer, 1997, pages 97–134.

[36] Cohn, A. G., Gooday, J. and Bennett, B.: "A comparison of structures in spatial and temporal logics", R. Casati, B. Smith and G. White (eds.) *Philosophy and the Cognitive Sciences : Proc. 16th Intl. Wittgenstein Symposium*, Holder-Pichler-Tempsky, Vienna 1994.

[37] Cohn, A. G. and Gotts, N. M.: "Spatial regions with undetermined boundaries", *Proc. Gaithesburg Workshop on GIS*, ACM, 1994.

[38] Cohn, A. G. and Gotts, N. M.: "A theory of spatial regions with indeterminate boundaries", C Eschenbach, C. Habel and B. Smith (eds.) *Topological foundations of Cognitive Science*, 1994.

[39] Cohn, A. G. and Gotts, N. M.: "The egg-yolk representation of regions with indeterminate boundaries", P. Burrough and A. M. Frank (eds.) *Proc. GISDATA Specialist Meeting on Geographical Entities with Undetermined Boundaries*, Francis Taylor, 1996, pages 171–187.

[40] Cohn, A. G. and Gotts, N. M.: "A mereological approach to representing spatial vagueness", J. Doyle, L. C. Aiello and S. Shapiro (eds.) *Principles of Knowledge Representation and Reasoning, Proc. 5th Intl. Conf. (KR'96)*, Morgan Kaufmann, 1996, pages 230–241.

[41] Cohn, A. G., Bennett, B., Gooday, J. and Gotts, N. M.: "RCC: a calculus for region based qualitative spatial reasoning", *Geoinformatica*, **1**, 1997, pages 275–316.

[42] Cohn, A. G., Randell, D. A. and Cui, Z.: "Taxonomies of Logically Defined Qualitative Spatial Relations", *International Journal of Human-Computer Studies: Special issue on Formal Ontology in Conceptual Analysis and Knowledge Representation*, **43**(5–6), 1995, pages 831–846.

[43] Cohn, A. G. and Varzi, A. C.: "Connection relations in mereotopology", H. Prade (ed.) *Proc. 13th European Conf. on AI (ECAI-98)*, J Wiley and Sons, 1998, pages 150-154

[44] Cohn, A. G. and Varzi, A. C.: "Modes of Connection: a taxonomy of qualitative topological relation", C. Freksa and D. M. Mark (eds.) *Spatial Information Theory: Cognitive and Computational Foundations of Geographic Information Systems, Proc. COSIT'99*, LNCS No. 1661, Springer-Verlag, 1999, pages 299–314.

[45] Cristani, M.: "The Complexity of Reasoning about Spatial Congruence", *Journal of Artificial Intelligence Research*, **11**, 1999, pages 361-390.

[46] Cristani, M., Cohn, A. G. and Bennett, B.: "Spatial Locations via Morpho-Mereology", A. G. Cohn and F. Giunchiglia and B. Selman (eds.) *Principles of Knowledge Representation and Reasoning: Proc. 7th Intl. Conf. (KR 2000)*, Morgan Kaufmann, 2000, pages 15–25.

[47] Cui, Z., Cohn, A. G. and Randell, D. A.: "Qualitative simulation based on a logical formalism of space and time", *Proc. American Conf. on AI (AAAI-92)*, 1992, pages 679–684.

[48] Davis, E.: "Solutions to a Paradox of Perception with Limited Acuity", R. Brachman, H. Levesque and R. Reiter (eds.) *Principles of Knowledge Representation and Reasoning: Proc. 1st Intl. Conf. (KR'89)*, Morgan Kaufmann, 1989, pages 79–82.

[49] Davis, E.: "Representations of Commonsense Knowledge", Morgan Kaufman,San Mateo, California, 1990.

[50] Davis, E., Gotts, N. M. and Cohn, A. G.: "Constraint networks of topological relations and convexity", *Constraints*, **4**(3), 1999, pages 241-280.

[51] de Laguna, T.: "Point, line and surface as sets of solids", *The Journal of Philosophy*, **19**, 1922, pages 449-461.

[52] Dornheim, C.: "Vergleichende anlyse topologischer ansaetze des qualitativen raeumlichen siessens", *Studienarbeit, fachereich informatik*, Universitaet Hamburg, 1995

[53] Düntsch, I. and Gediga, G.: "Uncertainty measures of rough set predictions", *Artificial Intelligence*, **106**, 1998, pages 109–137.

[54] Düntsch, I., Wang, H. and McCloskey, S.: "A relation-algebra approach to Region Connection Calculus", *Technical Report*, School of Information and Software Engineering, University of Ulster, 1998.

[55] Edwards, G.: "The Voronoi Model and Cultural Space: Applications to the Social Sciences and Humanities", A. Frank and I. Campari (eds.) *Spatial Information Theory: A theoretical basis for GIS, Proc. COSIT'93*, LNCS No. 716, Springer-Verlag, 1993, pages 202–214.

[56] Edwards, G.: "Reasoning about shape using the Tangential Axis Transform or the Shape's Grain", 1999. Available from web at `http: / / www. gmt. ulavalca/ homepages/ edwards/ paper_grain. html`

[57] Egenhofer, M. J.: "A formal definition of binary topological relationships", W. Litwin and H. Schek (eds.) *3rd Intl. Conf. on Foundations of Data Organization and Algorithms*, LNCS No. 367, 1989, pages 457–472.

[58] Egenhofer, M. J.: "Topological Similarity", *Proc. FISI workshop on the Toplogical Foundations of Cognitive Science*, Volume 37 of Reports of the Doctoral Series in Cognitive Science, University of Hamburg, 1994.

[59] Egenhofer, M. J.: "Query Processing in spatial-query-by-sketch", *Journal of Visual Languages and Computing*, **8**, 1997, pages 403-424.

[60] Egenhofer, M. J. and Frenzosa, R.: "Point-set Topological Spatial Relations", *International Journal of Geographical Information Syatems*, **5**(2), 1991, pages 161–174.

[61] Egenhofer, M. J. and Herring, J.: "Categorizing topological spatial relationships between point, line and area objects", *The 9-intersections: formalism and its use for natural language spatial predicates. Technical Report 94-1.*, National Centre for Geographic Information and Analysis, Santa Barbara, 1994.

[62] Egenhofer, M. J. and Al-Taha, K. K.: "Reasoning about Gradual Changes of Topological Relationships", A. U. Frank, I. Campari and U. Formentini (eds.) *Theories and Methods of Spatio-temporal Reasoning in Geographic Space*, LNCS No. 639, Springer-Verlag, 1992, pages 196–219.

[63] Egenhofer, M. J., Clementini, E, and Di Felice, P.: "Toplogical relations between regions with holes", *International Journal of Geographical Information Systems*, **8**(2), 1994, pages 129–144.

[64] Egenhofer, M. J. and Frenzosa, R.: "On the equivalence of topological relations", *International Journal of Geographical Information Systems*, **9**(2), 1995, pages 133–152

[65] Egenhofer, M. J. and Mark, D.: "Naive Geography", A. U. Frank and W. Kuhn (eds.) *Spatial Information Theory: A theoretical basis for GIS, Proc. COSIT'95*, LNCS No. 988, Springer-Verlag, 1995, pages 1–16.

[66] Egenhofer, M. J. and Sharma, J.: "Topological relationships between regions in R^2 and Z^2", D. Abel and B. C. Ooi (eds.) *3rd Int. Symposium on Large Spatial Databases*, LNCS No. 692, Springer-Verlag, 1993.

[67] Eschenbach, C. and Heydrich, W.: "Classical mereology and restricted domains", *International Journal of Human-Computer Studies*, **43**, 1995, pages 723–740.

[68] Escrig, M. T. and Toledo, F.: "A Framework Based on CLP Extended with CHRs for Reasoning with Qualitative Orientation and Positional Information", *Journal of Visual Languages and Computing*, **9**, 1998, pages 81-101.

[69] Escrig, M. T. and Toledo, F.: *Qualitative Spatial Reasoning : Theory and Practice - Application to Robot Navigation*. Frontiers in AI and Applications, Volume 47, IOS Press, Amsterdam, 1998.

[70] Evans, M. G. H.: "Can there be vague objects?", *Analysis*, **38**, 1978. Reprinted *Collected Papers*, Clarendon Press, Oxford, 1985

[71] Faltings, B.: "A symbolic approach to qualitative kinematics", *Artificial Intelligence*, **56**(2), 1992.

[72] Faltings, B.: "Qualitative spatial reasoning using algebraic topology", A. U. Frank and W. Kuhn (eds.) *Spatial Information Theory: A theoretical basis for GIS, Proc. COSIT'95*, LNCS No. 988, Springer-Verlag, 1995, pages 17-30.

[73] Fernyhough, J., Cohn, A.G. and Hogg, D.: "Constructing qualitative event models automatically from video input", *Image and Vision Computing*, **18**, 2000, pages 81–103.

[74] Fleck, M. M.: "The topology of boundaries", *Artificial Intelligence*, **80**(1), 1996, pages 1–27.

[75] Forbus, K.: "Qualitative reasoning about space and motion", D. Gentner and A. L. Stevens (eds.) *Mental Models*, Erlbaum, Hillsdale, NJ, 1983, pages 53–73.

[76] Forbus, K.: "Qualitative Process Theory", *Artificial Intelligence*, **24**, 1984, pages 85–168.

[77] Forbus, K., Nielson, P. and Faltings, B.: "Qualitative Kinematics : A framework", *Proc. Intl. Joint Conf. on AI (IJCAI-87)*, Milan, Italy, 1987, pages 430–436.

[78] Frank, A.: "Qualitative Spatial Reasoning about Distance and Directions in Geographic Space", *Journal of Visual Languages and Computing*, **3**, 1992, pages 343-373.

[79] Freksa, C.: "Temporal Reasoning based on semi-intervals", *Artificial Intelligence*, **54**, 1992, pages 199–227.

[80] Freksa, C. and Zimmermann, K.: "On Utilization of Spatial Structures for Cognitively Plausible and Efficient Reasoning", *Proc. IEEE Intl. Conf. on Systems, Man and Cybernetics*, 1992, page 18-21.

[81] Fujihara, H. and Mukerjee, A.: "Qualitative reasoning about document design", *Technical report*, Texas A and M University, 1991.

[82] Gahegan, M.: "Proximity operators for qualitative spatial reasoning", A. U. Frank and W. Kuhn (eds.) *Spatial Information Theory: A theoretical basis for GIS, Proc. COSIT'95*, LNCS No. 988, Springer-Verlag, 1995, pages 31-44.

[83] Galton, A. P.: "Towards an integrated logic of space, time and motion", *Proc. 13th Intl. Joint Conf. on AI (IJCAI-93)*, Chambery, France, 1993.

[84] Galton, A. P.: "Towards a Qualitative Theory of Movement", A. Frank and W. Kuhn (eds.) *Spatial Information Theory: A theoretical basis for GIS, Proc. COSIT'95*, LNCS No. 988, Springer, 1995, pages 377–396.

[85] Galton, A. P.: "Taking dimension seriously in qualitative spatial reasoning", W. Wahlster (ed.) *Proc. 12th European Conf. on AI (ECAI-96)*, John Wiley and Sons, 1996, pages 501-505.

[86] Galton, A. P.: "Space, Time and Movement", O. Stock (ed.) *Spatial and Temporal Reasoning*, Kluwer, 1997, pages 321–352.

[87] Galton, A. P.: "Modes of Overlap", *Journal of Visual Languages and Computing*, **9**, 1998, pages 61–79.

[88] Galton, A. P.: "The Mereotopology of Discrete Space", C. Freksa and D. M. Mark (eds.) *Spatial Information Theory - Cognitive and Computational Foundations of Geographic Information Science, Proc. COSIT'99*, LNCS No. 1661, Springer, 1999, pages 251-266.

[89] Gerevini, A. and Renz, J.: "Combining Topological and Qualitative Size Constraints for Spatial Reasoning", *Proc. 4th Intl. Conf. on Principles and Practice of Constraint Programming (CP'98)*, Springer-Verlag, 1998.

[90] Gerla, G.: "Pointless Geometries", F Buekenhout (ed.) *Handbook of Incidence Geometry*, Elsevier Science, B.V., 1995, pages 1015-1031.

[91] Giunchiglia, F. and Walsh, T.: "A theory of abstraction", *Artificial Intelligence*, **57**(2-3), 1992, pages 323-390.

[92] Glasgow, J., Narayanan, N. H. and Chandrasekaran, B.: *Diagrammatic Reasoning*, MIT Press, 1995.

[93] Gooday, J. M. and Cohn, A. G.: "Using spatial logic to describe visual languages", *Artificial Intelligence Review*, **10**(1-2), 1995. This paper also appears in P. MckEvitt (ed.) Integration of Natural Language and Vision Processing (Vol IV), Kluwer, 1996.

[94] Gooday, J. and Cohn, A. G.: "Transition-Based Qualitative Simulation", *Proc. 10th Intl. Workshop on Qualitative Reasoning*, AAAI press, 1996, pages 74–82.

[95] Gooday, J. M. and Cohn, A. G.: "Visual language syntax and semantics: A spatial logic approach", K. Marriott and B. Meyer (eds.) *Proc. AVI'96 Workshop on Theory of Visual Languages*, Gubbio, Italy, 1996.

[96] Goodman, J. and Pollack, R.: "Allowable sequences and order types in discrete and computational geometry", J. Pach (ed.) *New Trends in Discrete and Computational Geometry*, Springer-Verlag, 1993, pages 103-134.

[97] Gotts, N. M.: "Defining a 'doughnut' made difficult", C. Eschenbach, C. Habel and B. Smith (eds.) *Topological Foundations of Cognitive Science*, Volume 37 of Reports of the Doctoral programme in Cognitive Science. University of Hamburg, 1994.

[98] Gotts, N. M.: "How far can we 'C'? defining a doughnut using connection alone", J. Doyle, E. Sandewall and P. Torasso (eds.) *Principles of Knowledge Representation and Reasoning: Proc. 4th Intl. Conf. (KR'94)*, Morgan Kaufmann, 1994.

[99] Gotts, N. M.: "An axiomatic approach to topology for spatial information systsems", *Technical Report No. - 96.25*, School of Computer Studies, University of Leeds, 1996.

[100] Gotts, N. M.: "Formalising commonsense topology: The INCH calculus", *Proc. 4th Intl. Symposium on Artificial Intelligence and Mathematics*, 1996

[101] Gotts, N. M.: "Topology from a single primitive relation: defining topological properties and relations in terms of connection", *Technical Report No. - 96.23*, School of Computer Studies, University of Leeds, 1996.

[102] Gotts, N. M.: "Using the RCC formalism to describe the topology of spherical regions", *Technical Report No. - 96.24*, School of Computer Studies, University of Leeds, 1996.

[103] Gotts, N. M., Gooday, J. and Cohn, A. G.: "A connection based approach to commonsense topological description and reasoning", *The Monist*, **79**(1), 1996, pages 51-75.

[104] Grigni, M., Papadias, D. and Papadimitriou, C.: "Topological Inference", *Proc. Intl. Joint Conf on AI (IJCAI-95)*, Morgan Kaufmann, 1995, pages 901–906.

[105] Grzegorczyk, A.: "Undecidability of some topological theories", *Fundamenta Mathematicae*, **38**, 1951, pages 137–152.

[106] Guesgen, H. and Hertzberg, J.: "A constraint based approach to Spatio-Temporal reasoning", *Journal of Applied Intelligence*, **3**, 1992, pages 71–90.

[107] Haarslev, V.: "A fully formalized theory for describing visual notations", K. Marriott and B. Meyer (eds.) *Visual Language Theory*, Springer-Verlag, New York, 1998, pp. 261-292.

[108] Haarslev, V., Lutz, C. and Möller, R.: "Foundations of Spatioterminological Reasoning with Description Logics", A. G. Cohn, L. Schubert and S. Shapiro (eds.) *Principles of Knowledge Representation and Reasoning: Proc. 6th Intl. Conf. (KR'98)*, Morgan Kaufman, 1998, pages 112–124

[109] Hartley, R.: "A Uniform Representation for Time and Space and their Mutual Constraints", *Computers Math. Applications*, 1992.

[110] Hayes, P. J.: "The Naïve Physics Manifesto", D. Mitchie (ed.) *Expert Systems in the microelectronic age*, Edinburgh University Press, 1979.

[111] Hayes, P. J.: "Naïve Physics I : Ontology of Liquids", R. Hobbs and R. Moore (eds.) *Formal Theories of the Commonsense World*, Ablex Publishing Corporation, Norwood, NJ, 1985, pages 71-89.

[112] Hays, E.: "On defining motion verbs and spatial prepositions", Universitat des Saarlandes, 1989.

[113] Hernández, D.: "Qualitative Representation of Spatial Knowledge", *Lecture Notes in Artificial Intelligence No. 804*, Springer Verlag, 1994

[114] Hernández, D., Clementini, E. and Di Felice, P.: "Qualitative Distances", A. Frank and W. Kuhn (eds.) *Spatial Information Theory: A theoretical basis for GIS, Proc. COSIT'95*, LNCS No. 988, Springer, 1995, pages 45–58.

[115] Herskovits, A.: *Language and Spatial Cognition - An interdisciplinary study of prepositions in English*, Cambridge University Press, 1986.

[116] Hobbs, J.: "Granularity", *Proc. 9th Intl. Joint Conf. on AI (IJCAI-85)*, 1985, pages 432–435.

[117] Isli, A. and Cohn, A. G.: "An algebra for cyclic ordering of 2D orientations", *Proc. 15th American Conf. on AI (AAAI-98)*, 1998, pages 643–649.

[118] Isli, A. and Moratz, R.: "Qualitative spatial representation and reasoning: algebraic models for relative position", Fachbereich Informatik, Universitaet Hamburg, 1999.

[119] Jonsson, P. and Drakengren, T.: "A Complete Classification of Tractability in RCC-5", *Journal of Artificial Intelligence Research*, **6**, 1997, pages 211–221. http: / / www. cs. washington. edu/ research/ jair/ volume6/ jonsson97a. ps

[120] Jungert, E.: "Symbolic Spatial Reasoning on Object Shapes for Qualitative Matching", A. Frank and I. Campari (eds.) *Spatial Information Theory: A theoretical basis for GIS, Proc. COSIT'93*, LNCS No. 716, Springer-Verlag, 1993, pages 444–462.

[121] Kaufman, S.: "A formal theory of spatial reasoning", *Principles of Knowledge Representation and Reasoning: Proc. Intl. Conf. (KR'91)*, 1991, pages 347–356.

[122] Knauff, M., Rauh, R. and Renz, J.: "A Cognitive Assessment of Topological Spatial Relations: Results from an Empirical Investigation", *Spatial Information Theory: A theoretical Basis for GIS, Proc. COSIT'97*, LNCS No. 1329, Springer, 1997, pages 193–206.

[123] Knauff, M., Rauh, R. and Schlieder, C.: "Preferred mental models in qualitative spatial reasoning: A cognitive assessment of Allen's calculus", *Proc. of 17th Annual Conf. of the Cognitive Science Society*, Lawrence Erlbaum Associates, 1995, pages 200–205.

[124] Kuipers, B. and Byun, Y-T.: "A robot exploration and mapping strategy based on a semantic heirarchy of spatial representations", *Journal of Robotics and Autonomous Systems*, **8**, 1991, pages 47–63.

[125] Lehmann, F. and Cohn, A. G.: "The EGG/YOLK reliability heirarchy: Semantic data integration using sorts with prototypes", *Proc. Conf. on Information Knowledge Management*, ACM Press, 1994, pages 272–279.

[126] Leyton, M.: "A process grammar for shape", *Artificial Intelligence*, **34**, 1988, pages 213–247.

[127] Ligozat, G.: "Towards a General Characterization of Conceptual Neighbourhoods in Temporal and Spatial Reasoning", F. D. Anger and R. Loganantharah (eds.) *Proc. AAAI-94 Workshop on Spatial and Temporal Reasoning*, 1994.

[128] Ligozat, G.: "Reasoning about cardinal directions", *Journal of Visual Languages and Computing*, **9**, 1998, pages 23–44.

[129] Liu, J.: "A method of spatial reasoning based on qualitative trigonometry", *Artificial Intelligence*, **98**, 1998, pages 137–168.

[130] Lundell, M.: "A qualitative model of gradient flow in a spatially distributed parameter", *Proc. 9th Intl. Workshop on Qualtiative Reasoning (QR'95)*, Amsterdam, 1995, pages 131–138.

[131] Mark, D., Comas, D., Egenhofer, M. J., Freundschuh, S., Gould, J. and Nunes, J.: " Evaluating and refining computational models of spatial relations through cross-linguistic human-subjects testing", *Spatial Information Theory: A theoretical basis for GIS, Proc. COSIT'95*, LNCS No. 988, Springer, 1995, pages 553–568.

[132] Marr, D. and Nishihara, H. K.: "Representation and recognition of the spatial organization of three dimensional shapes", *Proc. Royal Society of London, Series B 200*, 1978, pages 269–294.

[133] Masolo, C. and Vieu, L.: "Atomicity vs. infinite divisibility of space", C. Freksa and D. M. Mark (eds.) *Spatial Information Theory - Cognitive and Computational Foundations of Geographic Information Science, Proc. COSIT'99*, LNCS No. 1661, Springer, 1999, pages 235–250.

[134] Mavrovouniotis, M. and Stephanopoulos, G.: "Formal order-of-magnitude reasoning in process engineering", *Computers and Chemical Engineering*, **12**, 1988, pages 867–881.

[135] Mukerjee, A.: "Neat vs Scruffy: A survey of Computational Models for Spatial Expressions", P. Oliver and K-P Gapp (eds.) *Representation and Processing of Spatial Expressions*, Kluwer, 1998.

[136] Mukerjee, A. and Joe, G.: "A qualitative model for space", *Proc. 8th American Conf. on AI (AAAI-90)*, Vol. **2**, 1990, pages 721–727.

[137] Muller, P.: "Éléments d'une théorie du mouvement pour la formalisation du raisonnement spatio-temporel de sens commun.", *PhD thesis*, Universite Paul Sabatier - Toulouse III, 1998.

[138] Muller, P.: "A qualitative theory of motion based on spatio-temporal primitives", A. G. Cohn, L. Schubert and S. Shapiro (eds.) *Principles of Knowledge Representation and Reasoning: Proc. 6th Intl. Conf. (KR'98)*, Morgan Kaufman, 1998, pages 131–141.

[139] Muller, P.: "Space-time as a primitive for space and motion", N. Guarino (ed.) *Formal ontology in Information Systems: Proc. 1st Int. Conf. (FOIS-98)*, Frontiers in AI and Applications, Volume 46, IOS Press, Amsterdam, 1998, pages 63–76.

[140] Nebel, B.: "Computational Properties of Qualitative Spatial Reasoning: First Results", I. Wachsmuth, C-R. Rollinger and W. Brauer (eds.) *KI-95: Advances in Artificial Intelligence*, LNCS No. 981, Springer-Verlag, 1995, pages 233–244.

[141] Nebel, B.: "Reasoning about temporal relations: a maximal tractable subset of Allen's interval algebra", *Journal of the Association for Computing Machinery*, **42**(1), 1995, pages 43–66.

[142] Pratt, I. and Schoop, D.: "A complete axion system for polygonal mereotopology of the plane", *Journal of Philosophical Logic*, **28**(6), 1998.

[143] Pratt, I. and Lemon, O.: "Ontologies for Plane, Polygonal Mereotopology", *Noctre Dame Journal of Formal Logic*, **38**(2), 1997, pages 225–245.

[144] Raiman, O.: "Order of Magnitude Reasoning", *Proc. American Conf. on AI (AAAI-86)*, 1986, pages 100–104.

[145] Rajagopalan, R.: "A model for integrated qualitative spatial and dynamic reasoning about physical systems", *Proc. American Conf. on AI (AAAI-94)*, 1994, pages 1411–1417.

[146] Rajagopalan, R. and Kuipers, B.: "Qualitative spatial reasoning about objects in motion: application to physics problem solving", *IEEE Conf. on AI for Applications*,San Antonio, TX, 1994.

[147] Ralha, C. G.: "A Framework for Dynamic Structuring of Information", *PhD dissertation*, School of Computer Studies, University of Leeds, 1996.

[148] Randell, D. A. and Cohn, A. G.: "Exploiting lattices in a theory of space and time", *Computers and Mathematics with Applications*, **23**(6-9), 1992, pages 459–476. Also appears in F Lehmann (ed.) *Semantic Networks*, Pergamon Press, Oxford, 1992, pages 459–476.

[149] Randell, D. A., Cohn, A. G. and Cui, Z.: "Computing Transitivity Tables: A challenge for automated theorem provers", *Proc. CADE 11*, Springer-Verlag, 1992.

[150] Randell, D. A., Cohn, A. G. and Cui, Z.: "Naive Topology: Modelling the force pump", P. Struss and B. Faltings (eds.) *Advances in Qualitative Physics*, MIT Press, 1992, pages 177–192.

[151] Randell, D. A., Cui, Z. and Cohn, A. G.: "A spatial logic based on region and connection", *Principles of Knowledge Representation and Reasoning: Proc. 3rd Intl. Conf. (KR'92)*, Morgan Kaufmann, 1992, pages 165–176.

[152] Randell, D. A. and Cohn, A. G.: "Modelling topologocal and metrical properties of physical processes", R. Brachman, H. Levesque and R. Reiter (eds.) *Principles of Knowledge Representation and Reasoning: Proc. 1st Intl. Conf. (KR'89)*, Morgan Kaufmann, 1989, pages 55–66.

[153] Renz, J.: "Maximal Tractable Fragments of the Region Connection Calculus", *Proc. 16th Intl. Joint Conf. on AI (IJCAI-99)*, 1999, pages 448–454.

[154] Renz, J. and Nebel, B.: "A canonical model of the Region Connection Calculus", A. G. Cohn, L. K. Schubert and S. Shapiro (eds.) *Principles of Knowledge Representation and Reasoning: Proc. 6th Intl. Conf. (KR-98)*, Morgan Kaufmann, 1998, pages 330–341.

[155] Renz, J. and Nebel, B.: "On the complexity of qualitative spatial reasoning: A maximal tractable fragment of the Region Connection Calculus", *Artificial Intelligence*, **108**(1-2), 1999, pages 69–123.

[156] Requicha, A. G. and Boelcke, H. B.: "Solid Modelling: a historical summary and a contemporary assessment", *IEEE Computer Graphics and Applications*, **2**, 1992, pages 9–24.

[157] Richards, W. and Hoffman, D. D.: "Codon constraints on closed 2D shapes", *Computer Vision, Graphics and Image Processing*, **31**, 1985, pages 265–281.

[158] Röhrig, R.: "A theory of Qualitative Spatial Reasoning based on order relations", *Proc. 12th American Conf. on AI (AAAI-94)*, 1994, pages 1418–1423.

[159] Sablayrolles, P.: "Spatio-temporal semantics in natural language: the case of motion", M. Aurnague et al. (eds.) *Semantics of Time, Space, Movement and Spatio-Temporal Reasoning*, Working Papers of the 4th Intl. Workshop, 1992, pages 69–88.

[160] Schlieder, C.: "Representing Visible Locations for Qualitative Navigation", N. Piera Carreté and M. G. Singh (eds.) *Qualitative Reasoning and Decision Technologies*, CIMNE, 1993, pages 523–532.

[161] Schlieder, C.: "Reasoning about ordering", A. U. Frank and W. Kuhn (eds.) *Spatial Information Theory - A Theoretical Basis for GIS, Proc. COSIT'95*, LNCS No. 988, Springer, 1995, pages 341–349.

[162] Schlieder, C.: "Qualitative Shape Representation", P. Burrough and A. M. Frank (eds.) *Proc. GISDATA Specialist Meeting on Geographical Objects with Undetermined Boundaries*, Francis Taylor, 1996.

[163] Shanahan, M.: "Default reasoning about spatial occupancy", *Artificial Intelligence*, **74**, 1995, pages 147–163.

[164] Simons, P.: *Parts: A study in Ontology*, Clarendon Press, Oxford, 1987.

[165] Sklansky, J.: "Measuring Concavity on a Rectangular Mosaic", *IEEE Transactions on Computers*, **C-21**(12), 1972, pages 1355–1364.

[166] Smith, B.: "Ontology and the logicistic analysis of reality", N. Guarino and R. Poli (eds.) *Proc. Intl. Workshop on Formal Ontology in Conceptual Analysis and Knowledge representation*, 1993, pages 51–68.

[167] Smith, B.: "Boundaries: An Essay in Mereotopology", L. H. Hahn (ed.) *Library of Living Philosophers*, Open Court, Chicago and La Salle, 1997, pages 534–561.

[168] Smith, B. and Varzi, A. C.: "The Niche", *Noûs*, **33**(2), 1999, pages 198–222.

[169] Sogo, T., Ishiguro, H. and Ishida, T.: "Acquisition of Qualitative Spatial Representation by Visual Observation", *Proc. 16th Intl. Joint Conf. on AI (IJCAI-99)*, 1999, pages 1054–1060.

[170] Stell, J. G. and Worboys, M. F.: "The algebraic structure of sets of regions", S. C. Hirtle and A. U. Frank (eds.) *Spatial Information Theory: A theoretical basis for GIS, Proc. COSIT'97*, LNCS No. 1329, Springer, 1997, pages 163–174.

[171] Tarski ,A.: "Foundations of the Geometry of Solids", *Logic, Semantics, Metamathematics*, Chapter 2, Clarendon Press, Oxford, 1956.

[172] Tate, A., Hendler, J. and Drummond, M.: "A review of AI planning techniques", J. Allen, J. Hendler and A. Tate (eds.) *Readings in Planning*, Morgan Kaufman, San Mateo, CA, 1990.

[173] Topaloglou, T.: "First order theories of approximate space", F. Anger et al. (eds.) *Proc. AAAI-94 workshop on spatial and temporal reasoning*, Seattle, 1994, pages 283–296.

[174] Varzi, A.C.: "Parts, Wholes and Part-Whole relations: the prospects of mereotopology", *Data and Knowledge Engineering*, **20**(3), 1996, pages 259–286.

[175] Varzi, A. C.: "On the boundary between mereology and topology", R. Casati, B. Smith and G. white (eds.) *Philosophy and the Cognitive Sciences: Proc. 16th Intl. Wittgenstein Symposium*, Hölder-Pichler-Tempsky, Vienna, 1994.

[176] Vieu, L.: "Sémantique des relations spatiales et inférences spatio-temporelles", *PhD dissertation*, Université Paul Sabatier, Toulouse, 1991.

[177] Vieu, L.: "Spatial Representation and Reasoning in AI", O. Stock (eds.) *Spatial and Temporal Reasoning*, Kluwer, 1997, pages 3–40.

[178] Vilain, M. and Kautz, H.: "Constraint Propogation Algorithm for Temporal Reasoning", *Proc. 5th American Conf. on AI (AAAI-86)*, Philadelphia, 1986, pages 377–382.

[179] Walischewski, H.: "Learning regions of interest in postal automation", *Proc. 5th Intl. Conf. on Document Analysis and Recognition (ICDAR'99)*, Bangalore, India, 1999.

[180] Weld, D. S. and De Kleer, J. (eds.) *Reading in Qualitative Reasoning About Physical Systems*, Morgan Kaufman, San Mateo, Ca, 1990.

[181] Whitehead, A. N.: D. R. Griffin and D. W. Sherburne (eds.) *Process and Reality: Corrected edition*, The Free Press, MacMillan Pub. Co., New York, 1978.

[182] Wolter, F. and Zakharyaschev, M.: "Spatio-temporal representation and reasoning based on RCC-8", A. G. Cohn, F. Giunchiglia and B. Selman (eds.) *Principles of Knowledge Representation and Reasoning: Proc. 7th Int. Conf. (KR 2000)*, 2000, pages 3–14.

[183] Woodger, J. H.: *The Axiomatic Method in Biology*, Cambridge University Press, 1937.

[184] Worboys, M. F. and Bofakos, P.: "A canonical model for a class of real spatial objects", D. Abel and B. C. Ooi (eds.) *Advances in Spatial Databases: 3rd Intl. Symposium (SSD'93)*, LNCS No. 692, Springer-Verlag, 1993, pages 36–52.

[185] Zimmermann, K.: "Enhancing qualitative spatial reasoning – combining orientation and distance", A. Frank and I. Campari (eds.) *Spatial Information Theory: A theoretical basis for GIS, Proc. COSIT'93*, LNCS No. 716, Springer-Verlag, 1993, pages 69–76.

[186] Zimmermann, K.: "Measuring without Distances: the delta calculus", A. Frank and W. Kuhn (eds.) *Spatial Information Theory: A theoretical basis for GIS, Proc. COSIT'95*, LNCS No. 988, Springer-Verlag, 1995, pages 59–68.

[187] Zimmermann, K. and Freksa, C.: "Enhancing Spatial Reasoning by the Concept of Motion", A. Sloman (ed.) *Prospects for Artificial Intelligence*, IOS Press, 1993, pages 140–147.