COT5520/CIS4930: COMPUTATIONAL GEOMETRY

Homework # 4

Due date: Dec 3, 2008, Thursday (beginning of the class)

Your solutions should be concise, but complete, and typed (or handwritten clearly). Feel free to consult textbooks, journal and conference papers and also each other, but write the solutions yourself and cite your sources. Answer **only four** of the following six questions. Each problem is worth 25 pts.

1. Let P be a finite set of points in the plane and H the dual set of lines. Translate the following statements about P to statements about H.
   (i) No three points of P lie on a common line.
   (ii) The point $p \in P$ is a vertex of the convex hull.
   (iii) P has a subset of k points in convex position.
   (iv) P is contained in a strip bounded by two parallel lines at unit distance from each other.

2. A k-*coloring* of a line arrangement is a map χ from the set of faces to $\{1, 2, \ldots, k\}$ such that $\chi(f) \neq \chi(g)$ if $f \neq g$ share a common edge.

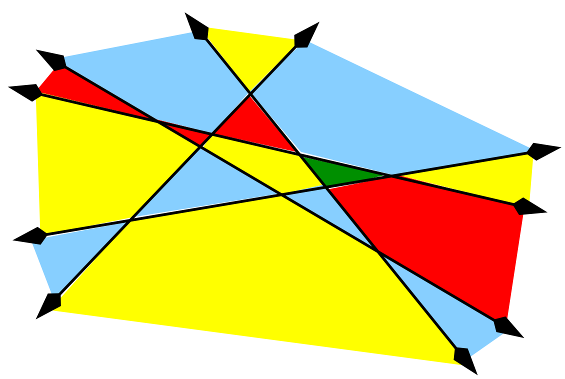

Figure 1: A 4-coloring of an arrangement of 5 lines.

   (i) What is the smallest k such that every line arrangement has a k-coloring?
   (ii) Which line arrangements are 2-colorable?
   (iii) Draw a 2-colorable line arrangement for which the ratio of the number of faces of one color over the number of faces of the other color is as large as you can manage. What ratio do you get?

3. Let H be a set of n lines in general position. Consider the graph whose nodes are the vertices and whose arcs are the bounded edges in the arrangement $\mathcal{A}(H)$. Describe an algorithm that takes time $O(n^2)$ and space $O(n)$ to compute the x-monotone path with the largest number of arcs.

4. Given n points in $\mathbb{R}^2$, define the optimal Euclidean Steiner tree to be a minimum length tree containing all n points and any other subset of points from $\mathbb{R}^2$. (General Steiner tree problem has long history and has been studied by well-known mathematicians including Fermat and Gauss.) Prove that each of the additional points in an Euclidean Steiner tree must have degree three, with all three angles being 120°.

5. Let G be a complete undirected graph in which all edge lengths are either 1 or 2. Give a 4/3 factor approximation algorithm for TSP in this special class of graphs.

6. Consider variants on the metric TSP problem in which the object is to find a simple path containing all the vertices of the graph. Three different problems arise, depending on the number (0, 1, or 2) of endpoints of the path that are specified. Obtain the following approximation algorithms:

   (a) If zero or one endpoints are specified, obtain a 3/2 factor approximation algorithm.

   (b) If both endpoints are specified, obtain a 5/3 factor approximation algorithm.