Preliminary Quiz

Please answer all the questions.

- Given a set S of n distinct integers,

  1. describe an algorithm to compute the second largest integer in S.
  2. describe an algorithm to compute the median of S. For simplicity, assume n is an odd integer. Then, note that the median of S is the number in S that is smaller than exactly half the numbers in S.

- In probability theory, the **birthday paradox** states that in a group of 23 (or more) randomly chosen people, there is more than 50% probability that some pair of them will have the same birthday. For 57 or more people, the probability is more than 99%, although it cannot be exactly 100% unless there are at least 366 people.

  In order to test this theory, we design an algorithm to check whether there exists two students in a class that have the same birthday. We suggest two algorithms:

  – For every pair of students in the class, I check whether the two students have the same birthday.
  – I pass a calendar around, I ask every student to mark his/her birthday on the calendar and let me know if his/her birthday is already marked by another student.

  1. Discuss which one of these algorithms take less time?
  2. Discuss which one of these algorithms use less memory space?

- Consider N the universe of firstnames. For the sake of simplicity, assume that each firstname is either a male name (like John, Patrick etc.) or a female name (like Sara, Sally etc.) – it cannot be an unisex name. Let C be the class of all sets which contain only male firstnames or contain only female firstnames. Let C also include the empty set.

  – Is the union of any pair of sets in C also in C?
  – Is the intersection of any pair of sets in C also in C?
  – Is the complement of any set in C also in C?

  I.e, is C closed under the operations of intersections, unions and complement respectively?

- Suppose the following sentence is true.

  > "If I attended all the lectures, I will get an A in this course."

  What can I conclude logically from the each of the following 2 cases:

  – I got an A.
  – I did not get an A.

- How many distinct subsets are there of the set $\{1, 2, 3, ..., n\}$? Your expression should be in terms of n.