COT5405: ANALYSIS OF ALGORITHMS

MidTerm Exam II

Date: Nov 1, 2007, Thursday
Time: 1:55pm – 3:55pm
Professor: Alper Üngör (Office CSE 430)

This is a closed book exam. No collaborations are allowed. Your solutions should be concise, but complete, and handwritten clearly. Use only the space provided in this booklet, including the even numbered pages. Feel free to give reference to the algorithms, definitions and concepts discussed in class rather than describing them in detail. There are no bonus points.

GOOD LUCK!

Your name:

	Credit	Max
Problem 1		20
Problem 2		20
Problem 3		20
Problem 4		20
Problem 5		20
Total		100

1. [20 points = 4+4+4+4+4] TRUE/FALSE QUESTIONS (NO NEED FOR JUSTIFICATION)

   (a) TRUE/FALSE
   Bellman-Ford algorithm presented in class for computing shorthest paths is a *dynamic programming* algorithm.

   (b) TRUE/FALSE
   Dijkstra's algorithm determines whether a *negative cycle* exists in a graph.

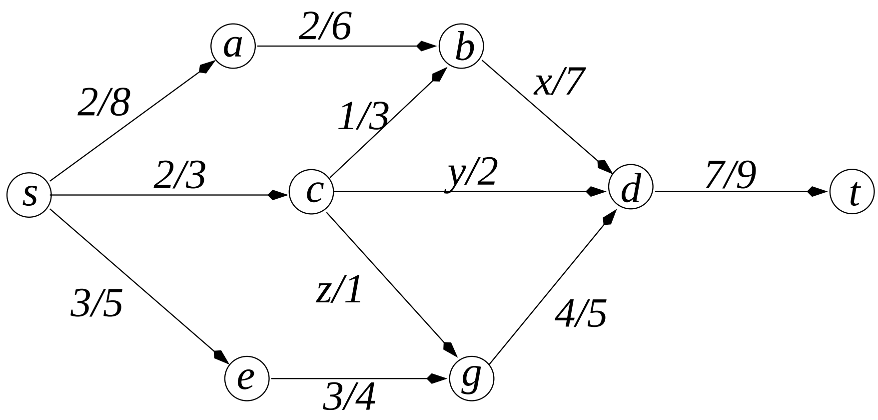
   (c) TRUE/FALSE
   Fractional cascading is a method that uses similarities between substructures to speed up searches in Range Trees.

   (d) TRUE/FALSE
   Let $G = (V, E)$ be a directed graph with weights on edges, and $\gamma(p, q)$ denote the length of the *longest* simple path between p and q. Then, we have the *triangle inequality*, i.e., $\gamma(p, q) + \gamma(q, r) \geq \gamma(p, r)$ for every p, q, and r in V.

   (e) TRUE/FALSE
   Consider the following pseudocode which describes a *greedy* algorithm that takes an undirected graph $G = (V, E)$ and a weight function w on its edges as input and returns a set of edges T. The output T of the algorithm is a minimum spanning tree of G.

```
MAYBE-MST(G, w)
Sort the edges of G into nondecreasing order of edge weights w
T ← E
for each edge e, taken in nondecreasing order by weight
        do if T − {e} is a connected graph
             then T ← T − e
return T
```

2. [20 points = 3 + 3 + 5 + 4 + 5] MAXIMUM FLOW
The Figure shows a flow network on which an $s-t$ flow has been assigned. The two numbers on each edge shows the flow and the capacity values, respectively. For instance, the edge ab carries a flow of 2, while its capacity is 6.



(a) What are the values of x, y, and z that make the assigned flow feasible?

(b) What is the value of this flow? Is this a maximum $s-t$ flow in this network?

(c) Draw the residual graph for this flow.

(d) Find a minimum $s-t$ cut in this network. What is the capacity of this minimum cut?

(e) Start with a **zero flow** in this flow network, and illustrate that the number of augmentations performed by the Edmonds-Karp algorithm can be more than the number of augmentations performed by the Ford-Fulkerson algorithm. (Note that this is counterintutive, as we have have established an upper bound of $O(VE)$ on the number of augmentations performed by the Edmods-Karp algorithm, but showed that no such bound exists for the Ford-Fulkerson algorithm.)

3. [20 points = 10 + 10] AMORTIZED ANALYSIS
Recall the amortized analysis of a sequence of n insertions into a dynamic tables whose capacity is doubled every time it becomes full. We used three different techniques in class to show that the amortized cost of each operation is $O(1)$. Here, we consider a modified version of the dynamic table.

(a) Instead of doubling the table, suppose we increase its capacity by only 50% everytime it becomes full. That is, the table capacity goes from C to $3C/2$, when it is full. Consider a sequence of n insertions into this table. Prove (using aggregate, accounting, or potential method) that the amortized cost of each insertion is $O(1)$.

(b) In addition to the insertion operation and the expansion rule described in part (a), suppose we also implement a deletion operation. Moreover, after a deletion, whenever less than two thirds of the table is full, we shrink the table capacity from C to $2C/3$. Consider a sequence of n operations (an arbitrary mix of insertions and deletions). Argue that the amortized cost of each operation can be $\Omega(n)$.

4. [20=10+10 points] MINIMUM SPANNING TREES/SHORTEST PATHS
Given two graphs G and G' that have the same sets of vertices V and edges E, however different weight functions (w and w' respectively) on their edges. Suppose for each graph the weights on the edges are distinct and satisfy the following relation: $w'(e) = w(e)^2$ for every edge e of E. Decide whether each of the following stements is true. Give either a short proof or a counter example.

(a) The minimum spanning tree of G is the same as the minimum spanning tree of G'.

(b) For a pair of vertices a and b in V, a shortest path between them in G is also a shortest path in G'.

5. [20 points = 10 + 10] COMPUTATIONAL GEOMETRY
In this problem we want to determine whether a given sequence $< p_0, p_1, \ldots, p_{n-1} >$ of n points froms the consecutive vertices of a convex polygon.

(a) Consider an algorithm that applies the orientation test at every vertex to see if it is a convex vertex (counterclockwise turn). Then, the algorithm concludes that the sequence forms a convex hull if and only if all vertices are convex. Argue that this algorithm takes $O(n)$ time, however, does not always produce a correct answer.

(b) Modify the algorithm given in part (a) so that it always produces a correct answer in $O(n)$ time.