

Solutions

COT 4501 Pretest (Spring 2009, Anand Rangarajan)

Please write your name, email address and UF ID at the top of your answer sheet. The test will carry no points towards your final grade. It is meant to give you feedback on how well you are prepared for this class. The graded tests will be returned in the next class. You have **25 minutes** to take this test. Please **budget your time** so that you spend no more than an average of 5 minutes on each question. Use the space provided to write your answers. No explanations necessary for the answers, unless you are specifically asked for one.

1. Limits: a)  $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$ , b)  $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$  [Hint: Use L'Hopital's rule.]

$\lim_{x \rightarrow 0} \frac{e^x}{1} = 1$        $\lim_{x \rightarrow 0} \frac{\cos x}{1} = 1$

2. Differentiation: a)  $\frac{d}{dx} x^n$ , b)  $\frac{d}{dx} n^x$ , c)  $\frac{d}{dx} x^x$  [Hint: Consider taking logarithms on both sides for parts b) and c).]

(a)  $n x^{n-1}$       (b)  $n^n \log n$

$\log y = x \log n$   
 $\frac{1}{y} \frac{dy}{dx} = x \left(\frac{1}{n}\right) + \log n$   
 $\frac{dy}{dx} = x^n (1 + \log n)$  (c)

3. Integration: a)  $\int x^y dx$ , b)  $\int y^x dx$  [Hint: For b), use the fact that  $\int e^x dx = e^x + c$  where  $c$  is an arbitrary constant.]

(a)  $\frac{x^{y+1}}{y+1} + c$       (b)  $\int e^{x \log y} dx = \frac{e^{x \log y}}{\log y} + c = \frac{y^x}{\log y} + c$

4. Minimization: You are given the fact that the maximum of  $x^{\frac{1}{x}}$  occurs at  $x = e$ . Determine whether  $\pi^e > e^\pi$  or  $e^\pi > \pi^e$  where  $e = \exp(1)$ . [Hint: Rewrite the relationship  $\pi^e \geq e^\pi$  by taking both sides to the power of  $\frac{1}{\pi e}$ .] Explain.

$\therefore x^{\frac{1}{x}}$  has max at  $x = e \therefore e^{\frac{1}{e}} > \pi^{\frac{1}{\pi}}$   
 $\Rightarrow \frac{1}{e} \log e > \frac{1}{\pi} \log \pi$   
 $\Rightarrow \frac{\log e}{e} > \frac{\log \pi}{\pi}$  or  $e^\pi > \pi^e$   $\therefore \log$  is monotonic

5. Optimization: Given a continued fraction expansion  $x = 2 + \frac{1}{2 + \frac{1}{2 + \dots}}$  what is the limiting value of  $x$ ? [Hint: Consider the limit to be  $x = 2 + \frac{1}{x}$ .]

$x = 2 + \frac{1}{x}$   
 $x - \frac{1}{x} = 2$   
 $x^2 - 1 = 2x$   
 $x = 1 \pm \sqrt{2}$   
 $\therefore x > 2 \therefore x = 1 + \sqrt{2}$