Notes for a Natural Cubic B-Spline

Assume that you have 7 (corresponding to $n+1$) data points $\{f(x_0), f(x_1), \ldots, f(x_6)\}$ and you need 9 (corresponding to $n+3$) coefficients $\{c_{-1}, c_0, c_1, \ldots, c_9\}$. From the natural spline endpoint constraints, you get $c_{-1} = 2c_0 - c_1$ and $c_7 = 2c_6 - c_5$. Using these relations, you can show that $c_0 = \frac{1}{6}f(x_0)$ and $c_6 = \frac{1}{6}f(x_6)$. This reduces the number of coefficients we seek to 5 (corresponding to $n-1$) since c_0 and c_7 can be obtained from the above two relations. When we substitute this information into equation (4.45) in the book, we get

$$\begin{bmatrix} 4 & 1 & 0 & 0 & 0 \\ 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} = \begin{bmatrix} f(x_1) - \frac{1}{6}f(x_0) \\ f(x_2) \\ f(x_3) \\ f(x_4) \\ f(x_5) - \frac{1}{6}f(x_6) \end{bmatrix}$$

which is of the form $Ac = b$ where A is 5×5, c is 5×1 and b is 5×1. The solution for the unknown coefficients is $c = A^{-1}b$ which can be efficiently computed in MATLAB using $A\backslash b$ which is equivalent to $A^{-1}b$. Once c is obtained, we can obtain c_{-1} and c_7 from the known solutions for c_0 and c_6.