# COT4501sp09

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HW 10 solutions

1. [5.3.4] $h = \frac{1}{4}$ So there are 4 intervals. So we group each of the two consecutive intervals into one and apply simpson's rule on each of the two subgroups formed and then add them. This is done in accordance with Eqn (5.5) on page263.

   So $f(x) = \frac{1}{1+x^3}$

   and on applying Simpson's rule we get the approximate value of the integral as:

   $S_4(f) = \frac{h}{3}(f(0) + 4f(\frac{1}{4}) + f(\frac{1}{2})) + \frac{h}{3}(f(\frac{1}{2}) + 4f(\frac{3}{4}) + f(1))$

2. [5.3.8] The best way is to use matlab for all problems of simpson's rule. That way you can experiment on the values of h you need to tale to get the desired accuracy. If you use $h = 1$in this case then the simpson's rule will be :

   $S_2(f) = \frac{h}{3}(f(-1) + 4f(0) + f(1))$ where $f(x) = \surd(1 - x^2)$

   see the matlab solutions for the above and this question for the answers and to see how the desired accuracy is reached and in how many steps.

3. [5.8.10] see the matlab solution.

4. [5.8.12] Here you have to derive the simpson's rule from the vary basic that is starting from what is given from Page 261. The only difference is that in there, $c - a = h$ and $b - c = h$. In here you have to use: $c - a = h$ and $b - c = \theta h$. Now derive the values of A, B and C. Remember their final values will only be in terms of $h$ and $\theta$. There will be no a, b and c in the final answer.

   So, $A = \int_a^{a+(\theta+1)h} \frac{(x-c)(x-b)}{(-h)(-(\theta+1)h)} dx$

   $= \int_a^{a+(\theta+1)h} \frac{(x-(a+h))(x-(a+(\theta+1)h))}{(-h)(-(\theta+1)h)} dx$

   $= \frac{1}{(h^2(\theta+1))} \int_{-h}^{\theta h} (u(u - \theta h))du$ (substituting $x - a - h = u$)

   Similarly, $B = \int_a^{a+(\theta+1)h} \frac{(x-a)(x-b)}{(h)(-\theta h)} dx$

$= -\frac{1}{\theta h^2}\int_a^{a+(\theta+1)h}(x-a)(x-(a+(\theta+1)h))dx$

$= -\frac{1}{\theta h^2}\int_0^{(\theta+1)h}(u)(u-(\theta+1)h)dx$ (substituting $x-a=u$)

And similarly, $C = \int_a^{a+(\theta+1)h}\frac{(x-a)(x-c)}{((\theta+1)h)(\theta h)}dx$

$= \frac{1}{\theta(\theta+1)h^2}\int_a^{a+(\theta+1)h}(x-a)(x-a-h)dx$

$= \frac{1}{\theta(\theta+1)h^2}\int_0^{(\theta+1)h}u(u-h)du$ (substituting $x-a=u$)

The integration above in all the three is trivial and you can do it without my simplifying further.

5. [5.8.14] refer page 265. The proof is similar.

   Let $q_k(x) = Ax^k + q_n(x)$ where, $q_k(x)$ is a polynomial of degree one greater than n where n is the degree (not the degree of precision) of the quadrature and $q_n(x)$ is a polynomial of degree n. from this it is clear that the quadrature rule will exactly integrate $q_n(x)$. However, it will exactly integrate $q_k(x)$ if it exactly integrates $x^k$. If it is so then the "degree of precision" of the quadrature is k. Similarly if this holds then we can keep going higher one step at a time i.e., now we will see whether our quadrature rule exactly integrates $q_{k+1}(x) = Bx^{k+1} + q_k(x)$. Now we are checking this step only if our quadrature rule exactly integrated $q_k(x)$. So we can say that similarly as argued above $q_{k+1}(x)$ will be exactly integrated if we can integrate $x^{k+1}$ exactly. Now we will continue this step $\forall 0 \leq k \leq p$. And so if we see that all these terms i.e., $x^k$ terms are exactly integrated $\forall 0 \leq k \leq p$, then this implies that the quadrature rule has the degree of precision p.

   Q.E.D.