

**COT3100**

**Quiz 2 February 16, 2009**

**Name:**

**Write your name and solutions clearly on this page (and back if necessary).  
There are 10 points for this quiz.**

**Problem 1. (2 pts)** Write down the power set of ( $\emptyset$  denotes the empty set)

$$A = \{1, \{1, \emptyset\}, \emptyset\}.$$

**Solution:** The power set contains  $2^3 = 8$  elements. The set is

$$\{\emptyset, \{1\}, \{\{1, \emptyset\}\}, \{\emptyset\}, \{1, \{1, \emptyset\}\}, \{1, \emptyset\}, \{\{1, \emptyset\}, \emptyset\}, \{1, \{1, \emptyset\}, \emptyset\}\}.$$

**Problem 2. (4 pts)** Let  $A, B$  and  $C$  denote sets. Prove that

$$(A - B) - C = (A - C) - (B - C).$$

**Solution:** Note that  $A - B = A \cap \bar{B}$ . Therefore, the left-hand side is

$$(A \cap \bar{B}) \cap \bar{C} = A \cap \bar{B} \cap \bar{C}.$$

Now we need to show the right-hand side also equals to this.

$$(A - C) - (B - C) = (A - C) \cap \overline{(B - C)} = (A - C) \cap \overline{(B \cap \bar{C})} = (A \cap \bar{C}) \cap (\bar{B} \cup C).$$

by De Morgan's law. Now use the distributive law:

$$(A \cap \bar{C}) \cap (\bar{B} \cup C) = (A \cap \bar{C} \cap \bar{B}) \cup (A \cap \bar{C} \cap C) = (A \cap \bar{C} \cap \bar{B})$$

because  $(A \cap \bar{C} \cap C) = \emptyset$ . This shows that the left-hand side is the same as the right-hand side.

**Problem 3. (4 pts)** Let  $f(x) = ax + b$  and  $g(x) = cx + d$  be two functions from the set of real numbers to the set of real numbers, where  $a, b, c$  and  $d$  are constants. Determine the relation between the constants if we have  $f \circ g = g \circ f$ . (Hint: the

relation is  $ad + b = cb + d$ . You have to show why it is so).

**Solution:**

$$f \circ g = f(g(x)) = a(cx + d) + b = acx + ad + b$$

$$g \circ f = g(f(x)) = c(ax + b) + d = acx + cb + d$$

Therefore, if  $f \circ g = g \circ f$ , we must have  $ad + b = cb + d$ !