

Your name: \_\_\_\_\_

Score: \_\_\_\_\_

COT3100: Discrete Structures – Spring 2007 – Exam II – Sample Test

This is a closed book exam. No collaborations are allowed. Your solutions should be concise, complete, and legible. Use only the space provided below each question. **Good luck!**

1. [20 pts] Determine whether each of the following statements is TRUE or FALSE

(a) Greedy Algorithm solves the Halting Problem.

(b)  $x^6 + 6x + 66$  is  $O(x^6)$ .

(c)  $x^7$  is  $\Omega(x^6)$ .

(d)  $2x + 1$  is  $\Theta(x^2)$ .

(e) Insertion sort algorithm has logarithmic time complexity.

2. [20 pts] Prove that 5 divides  $n^5 - n$  whenever  $n$  is a non-negative integer.

3. [20 pts] Let  $P(n)$  be the statement that a postage of  $n$  cents can be formed using just 3-cent stamps and 5-cent stamps. The parts of this question outline a strong induction proof that  $P(n)$  is true for  $n \geq 8$ .

(a) Show that  $P(8)$ ,  $P(9)$ , and  $P(10)$  are true completing the basis step.

(b) What is the inductive hypothesis of the proof?

(c) What do you need to prove in the inductive step (assuming the inductive hypothesis)?

(d) Complete the inductive step for  $k \geq 11$ .

4. [20 pts] Give a recursive algorithm for finding the sum of the squares of the first  $n$  even positive integers.  $(2^2 + 4^2 + 6^2 + \dots + (2n)^2)$

5. [20 pts] A company stores products in a warehouse. Storage bins in this warehouse are specified by their aisle, location in the aisle, and shelf. There are 50 aisles, 85 horizontal locations in each aisle, and 5 shelves throughout the warehouse. What is the least number of products the company can have so that at least two products must be stored in the same bin?