Prove that in any triangulation of a convex n-gon ($n \geq 4$), there will be always atleast two non-adjacent vertices which are not the endpoints of any diagonal.

Proof
Recall: a triangulation of a convex n-gon is the division of the polygon into triangles by means of non-intersecting diagonals.
Base Case: We did this for the case of a quadrilateral. Join any one of the two diagonals of a quadrilateral and you see that the other two vertices are not endpoints of any diagonal in the triangulation.
Induction Hypothesis: Assume the said property holds true for all convex polygons with $4, 5, 6, ..., k$ vertices. We need to prove it for a convex polygon with $k + 1$ vertices.
Induction Step: Draw a $(k+1)$-gon and join any one of the diagonals (say $v_i v_j$) which will divide the convex $(k+1)$-gon into two convex polygons of m and $k+3-m$ vertices respectively (actually draw this on a piece of paper to get a better picture). Let us name these polygons as P_1 and P_2 respectively. Firstly, assume that both have 4 or more vertices. Construct a triangulation of P_1 and a triangulation of P_2. Now, in any triangulation of P_1, there will be two non-adjacent vertices which are not the endpoints of any diagonals (by the induction hypothesis). The same is true for P_2 as well. As the vertices with the said property are non-adjacent, it cannot be the case that both v_i and v_j are not endpoints of any diagonal in a triangulation of P_1 (or in a triangulation of P_2 as well). So atleast one vertex $w_1 \in P_1$ s.t. $w \neq v_i \wedge w \neq v_j$ is the endpoint of no diagonal. Likewise, at least one vertex $w_2 \in P_2$ s.t. $w_2 \neq v_i \wedge w_2 \neq v_j$ is the endpoint of no diagonal. Now let us merge the two triangulated polygons P_1 and P_2 to get back the original $(k + 1)$-gon, retaining the diagonal $v_i v_j$. Now clearly we have with us a triangulation of the $(k + 1)$-gon, and clearly w_1 and w_2 have no diagonal joining them. So the theorem is proved for this case.

Now what happens if P_1 happens to be a triangle? P_1 doesn't obey the property, but the number of vertices in P_2 is more than 3 but less than $k+1$ and by the induction hypothesis, P_2 does possess the said property. Let $w_2 \in P_2$ be a vertex of P_2 which is the endpoint of no diagonal, and clearly there exists a vertex w_2 which is unequal to v_i and v_j (just as we did in the previous paragraph). Now vertex w_2 from P_2 and the third vertex of P_1 (which is unequal to both v_i and v_j) are not endpoints of any diagonal, and the theorem goes through for this case as well.