

- Quiz 5, COT 3100
 - NAME:
 - SECTION NUMBER:
 - This is a take-home quiz due in class on 27th Nov.
 - **You may consult the course textbook for this quiz, but you are not allowed to collaborate with any student (or anybody else). Internet browsing for solutions is strictly forbidden. If plagiarism is detected, it will be dealt with very strictly.**
 - For every (sub-)question, explain briefly how you arrived at the answer. Just the direct answer without explanation will receive zero credit.
1. If I have an array of n distinct numbers, you know that the numbers can be arranged in $n!$ ways. Now if I add the constraint that the median of these numbers must lie in its correct place in every arrangement, how many arrangements do I have? How many arrangements of the array are possible such that the maximum element is in its correct place? How many arrangements of the array are possible such that the maximum element is in its correct place and the median is also in its correct place? **If the numbers in the array of n elements were not all distinct, i.e. there was a group of some k_1 numbers that were all equal to one another, and another group of k_2 numbers all equal to one another (but unequal to any number from the previous group of k_1 numbers), such that $k_1 + k_2 + 2 = n$, how many distinct arrangements of the array are there? (Forget the constraints) [3 points]**

2. Suppose I have 100 cards, with 25 each of the same color (say blue, red, green, black). How many cards (at a minimum) must be chosen in order to ensure that a minimum of 3 black cards will be always be part of the selection? To ensure that a minimum of 3 black cards and a minimum of 3 blue cards are always part of the selection? To ensure that a minimum of 2 cards of every color are always part of the selection? [**2 points**]

3. Imagine, there are $2n$ people sitting around an ellipse-shaped table at $2n$ different points on the border. Suppose I wish to draw line segments between the points where the people are sitting, in such a way that no two segments intersect each other, and every point has exactly one segment passing through it. You are asked to develop a recursive formula for $K(2n)$, where $K(2n)$ refers to the number of ways to draw line segments between $2n$ points on the border while obeying the constraints given. If there are just two people at the table, you see that there is just one way to do this ($K(2) = 1$), so that's your base case. To get a better feel for this, you should draw a diagram with $2n = 4$ and $2n = 6$ points. Hint: You might find it useful to apply the sum rule and product rule. **[5 points]**