Homework 9

The homework is due in class (during the first 10 minutes) on Tuesday, 20th November (or before). The problems are mostly from the book by Rosen, 6th edition (sections 5.1, 5.2, 5.3). As mentioned in the course syllabus, there are no electronic submissions and no late homeworks will be accepted unless you have an illness spanning the full period from the time the homework was assigned until it was due (and I shall need to see a medical practitioner's certificate to that effect). Standard academic honesty rules apply. You can discuss problems with one another, but the solutions turned in should be entirely your own. Cases of plagiarism will be dealt with strictly. **Also, make sure you write your name and section number on the first homework sheet and also staple all the sheets together. UNSTAPLED OR UNNAMED HOMEWORKS WILL NOT BE GRADED.**

1 Basics of Counting

Section (5.1): Problem 4, 16, 22, 34, 44.

2 Pigeonhole Principle

Section (5.2): 4, 16, 14(a) and (b), 25.

3 Permutations and Combinations

Section (5.3): 12, 24, 28, 32.

4 One more problem

Given the two-set inclusion exclusion principle that $|A \cup B| = |A| + |B| - |A \cap B|$, prove the following inclusion exclusion principle for three sets: $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

5 Still one more problem

This is a **geometric** problem, so I just couldn't resist :-)! Note that this is a compulsory problem. Imagine a regular hexagon with unit length of each side. Suppose you pick 7 points randomly from inside the hexagon. Prove that at least two of them will be at a distance of less than 1 from each other. Now suppose I sample (i.e. pick) 25 points from inside the hexagon all randomly. Prove that

that there exists at least one pair at a distance of less than half from one another. Now suppose I sample 97 points from inside the hexagon all randomly. Prove that there exists a pair at a distance of less than 1/4 from one another. In order to guarantee that there will exist a pair of points at a distance of less than $\frac{1}{2^k}$ from one another, how many points must I sample at a minimum? Recall that a hexagon is a polygon with 6 sides. A regular polygon has all angles equal to one another and all sides equal to one another in length. (A regular polygon is always convex. If you don't see this, think it through).

6 Optional Problems

1. Read Theorem 3 on page 351, 352. Also read example 13 on page 352. In section (5.2), try problem 36. In section (5.3), try problem 44.

2. Here are a few thought experiments that you should perform for the sake of clarifying your concepts: If I have a string of k unique characters, in how many ways can I shuffle it? If I have a string of k unique characters (non-numeric), followed by m unique digits, in how many ways can I shuffle it? If I have a string of k unique characters (non-numeric), followed by m unique digits, in how many ways can I shuffle it so that the digits and the characters do not mix? What kind(s) of task on such a string can be performed in $k! + m!$ ways? The aim behind all these exercises is to help you to understand the interplay between the sum and product rule.