Homework 1

August 30, 2007

The homework is due in class (during the first 10 minutes) on Thursday, 6th September (or before). The problems are from the book by Rosen, 6th edition (sections 1.1 and 1.2). As mentioned in the course syllabus, there are no electronic submissions and no late homeworks will be accepted unless you have an illness spanning the full period from the time the homework was assigned until it was due (and I shall need to see a medical practitioner's certificate to that effect). Standard academic honesty rules apply. You can discuss problems but the solutions turned in should be entirely your own. Cases of plagiarism will be dealt with strictly.

1 Question 1

Translate the following statements into propositional calculus. Assume the following propositions:
p: You get an A on the final exam.
q: You do your homework regularly.
r: You get an A in the class.

1. You get an A in the class, but you do not do your homework regularly.
2. To get an A in the class, it is necessary to get an A in the final.
3. Getting an A in the final and doing your homework regularly is sufficient for you to get an A in the class.
4. You can get an A in the class if and only if you either do your homework regularly or you get an A in the final exam (or both).

2 Question 2

State the converse, inverse and contrapositive of the following sentences:

1. If it snows today, I shall ski tomorrow.

2. I come to class if there is going to be a quiz.
3. A positive integer is prime **only if** it has no divisors other than itself and the number 1.

3 Question 3

Here is a logic puzzle. An explorer is captured by cannibals on an island. The cannibals are of two kinds - those that always tell the truth and those that always lie. The cannibals will barbecue the explorer unless he can determine whether a particular cannibal always tells lies or always tells the truth. In order to determine that, the explorer is allowed to ask the cannibal one and only one question. Explain why the question "Are you a liar?" does not work. Now find a question that the explorer can ask in order to determine whether a given cannibal is a liar or a truth-teller. There will be multiple solutions and you could give any one. The question should involve the propostion p which stands for "you are a liar". The question will be of the form "Is it true that ...?". Note, you cannot ask the cannibal a question such as "Is the sky blue?" or "Are you a cannibal?", because they do not involve the proposition we just defined.

4 Question 4

1. Prove that $\neg(p \oplus q) \equiv p \Leftrightarrow q$ with and without a truth table.
2. With and without a truth table, show that $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$ is a tautology.

5 Question 5

Prove that the following are **NOT** equivalent:

1. $(p \wedge q) \Rightarrow r$ and $(p \Rightarrow r) \wedge (q \Rightarrow r)$.
2. $(p \Rightarrow q) \Rightarrow r$ and $p \Rightarrow (q \Rightarrow r)$.

6 $\star$ Question 6

In class, we have seen how the negation operator ($\neg$), the conjunction operator ($\wedge$)and the disjunction operator ($\vee$) can be used to express all other operators in propositional calculus. Later on, with the help of DeMorgan's laws, we showed that we can make do with either just negation and conjunction, or with just negation and disjunction. Now suppose we define a new operator p NAND q which is true if either p or q or both are false, and false if both p and q are true. Write down the truth table for the NAND operator. Use the definition of the

NAND operator to express the negation operator. In other words, express the proposition $\neg p$ using only the NAND operator and of course the proposition p. Repeat this to express $p \wedge q$ and $p \vee q$ with only the NAND operator, and the variables p and q.

7 Other practice problems (no need to submit these)

1. Show that $p \Leftrightarrow q$ and $\neg p \Leftrightarrow \neg q$ are equivalent.

2. If p is "You have flu", q is "You miss the final" and r is "You pass the course", then translate the following into English: $p \Rightarrow q$, $\neg q \Leftrightarrow r$, $(p \wedge q) \vee (\neg q \vee r)$.

3. Write down a compound proposition involving p, q and r that is true when p and q are both true and r is false, but false otherwise. (HINT: use the conjunction of each variable or their negation). Write a compound proposition that is true when exactly two of the three variables are true, and is false otherwise. How about at least two instead of exactly two?