

## Particles & Shape Grammars

### Kinematics

*kinematics* – motion (trajectories only – without consideration of physical considerations such as forces, friction, inertia etc.) as opposed to *dynamics*.

angle  $\theta$ , position  $\mathbf{p}$

$$\mathbf{p} = F(\theta) \quad \text{(forward) kinematic equation}$$

$$\theta = F^{-1}(\mathbf{p}) \quad \text{inverse kinematic equation}$$

We need the set of angles  $\theta$  for animation.

Inverse kinematics is difficult since  $F^{-1}$  may not be well-defined (there may be several or no way to achieve position  $\mathbf{p}$ ).

Alternative: *keyframe animation* ('inbetweening' or 'interpolation')

### Newtonian Particles

position  $\mathbf{p}$ , velocity  $\mathbf{v}$ , state  $Y := \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \end{bmatrix} \in R^6$  as functions of time  $t$

$$Y' = \begin{bmatrix} \mathbf{p}' \\ \mathbf{v}' \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ \text{force/mass} \end{bmatrix} =: \mathbf{g}(Y, t)$$

To determine the state at time  $t + h$  from the state at time  $t$  need to solve the ODE.

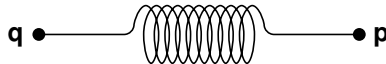
**Examples of force  $\mathbf{f}$ :** (1) *gravitational force*

$$\mathbf{f} = \begin{bmatrix} 0 \\ -\gamma \\ 0 \end{bmatrix}$$

(2) *spring force* (harmonic oscillator) between  $\mathbf{p}$  and  $\mathbf{q}$ .

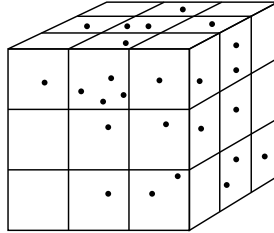
Define  $\mathbf{d} := \mathbf{p} - \mathbf{q}$  and  $d := \|\mathbf{d}\|$ . (Hooke's law + damping):

$$\mathbf{f} = -\left(k_{\text{spring}}(d - d_0) + k_{\text{damp}} \frac{\mathbf{d}' \cdot \mathbf{d}}{d}\right) \frac{\mathbf{d}}{d}.$$



repulsing force

$$\mathbf{f} = -\frac{k_{\text{repulse}}}{d^2} \frac{1}{d} (\mathbf{p} - \mathbf{q})$$



avoid  $O(n^2)$

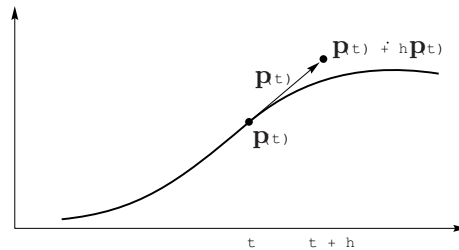
### Solving the ODE

Euler (should not be used! But explains basic forward-solving idea)

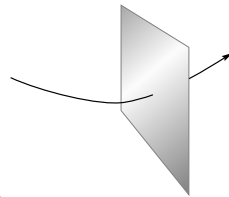
$$Y(t+h) = Y(t) + hY'(t) + O(h^2) = Y(t) + hg(Y, t) + O(h^2)$$

stiff ODE's, Runge Kutta solvers, Verlet Integration!

Implicit Euler:  $Y(t+h) = Y(t) + hg(Y(t+h), t+h)$



### Constraints



soft (approximate) and hard e.g. collision

- Detection — difficult! bounding boxes, quadtrees, binary space partitioning
- Reaction: Let  $\bar{\mathbf{p}} := \mathbf{p}(t) - \mathbf{p}(t^*)$  where  $t$  is the previous time and  $t^*$  the time of intersection,  $\mathbf{n}$  is the normal at  $\mathbf{p}(t^*)$ . Then the new point is in the direction

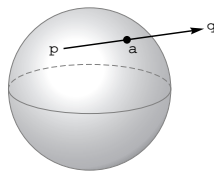
$$\mathbf{p} + 2(\bar{\mathbf{p}} - (\bar{\mathbf{p}} \cdot \mathbf{n})\mathbf{n}).$$

This corresponds to an *elastic* collision (no energy loss, deformation).

- Soft constraints with penalty function:  $\min \|\mathbf{p} - \mathbf{p}_0\|^2$ .

### Examples

- `bounce.c`
- sphere bouncing: intercept with sphere of radius  $r$




$$\|(1 - \lambda)\mathbf{p}(t) + \lambda\mathbf{p}(t + h)\| = r$$

Normal:  $\frac{(1-\lambda)\mathbf{p}(t) + \lambda\mathbf{p}(t+h)}{r}$ .

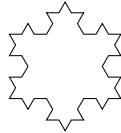
## Language based models

*tree, shape grammar*

Example (Koch curve, snowflake)  $F \rightarrow FLFRRLFLF$   $\longrightarrow$  

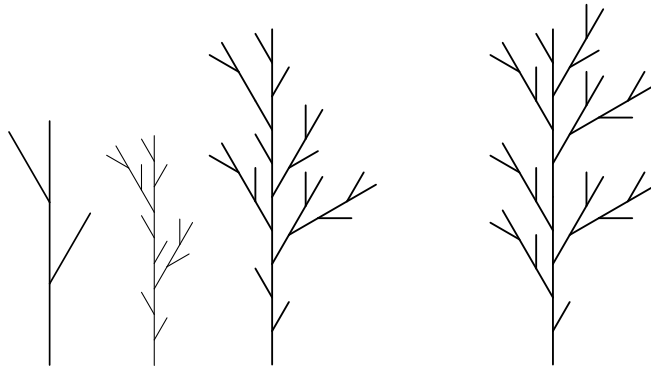


(a)



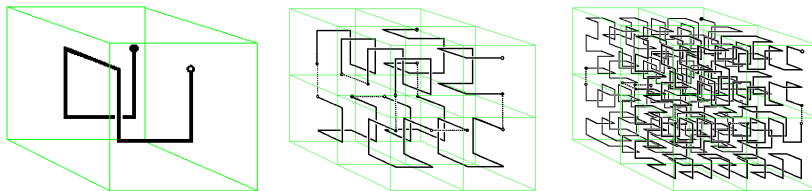
(b)

There exists a unit circle so that the Koch curve inside it has infinite length!



Example  $F \rightarrow F[RF]F[LF]F$  + random  
OPENGL/LRfrac

## Space filling Hilbert curve

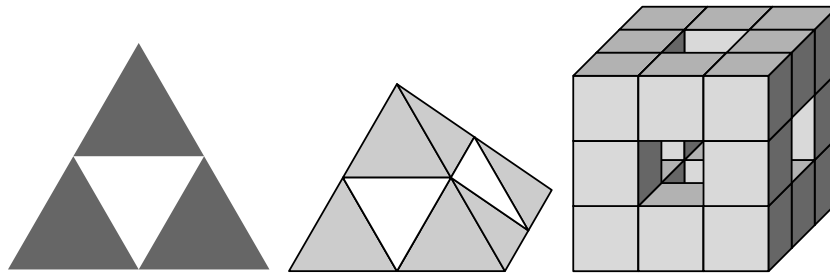


## Fractals

self-similar structures

the fractal dimension  $\text{dim}$  : pieces  $\cdot$  length<sup>dim</sup> = 1.

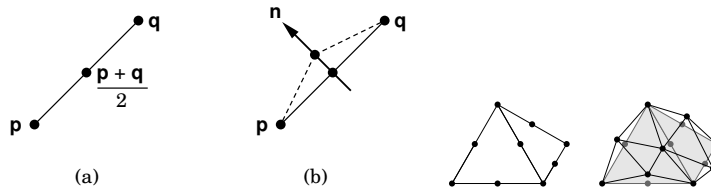
$$\text{dim} = \frac{\ln(\text{new}) \text{ pieces}}{\ln(\text{old edge}) \text{ length}}$$



$$\ln(20)/\ln(3) = 2.726833..$$

### Fractal mountains

brownian (random) “motion” (displacement) –  
 underlying Gaussian with variance: (length of old curve)<sup>2(2-dim)</sup> yields fractal dimension  $\text{dim}$ .



### Mandelbrot set

(see also: Julia set)

$$z_{k+1} = z_k^2 + c, \quad z_0 = 0, \quad z = x + iy \in C$$

Cex/Angel/mandelbrotcrash

Mandelbrot set == the set of all  $c$  for which this iteration converges.

