

Home Work 1: CAP 6610 Spring '07

Due Date: Feb 8th 2007

Show all steps. Be as concise as possible.

1. Prove that for any concave function Ψ , $\int \Psi(\Phi(x))dx \leq \Psi(\int \Phi(x)dx)$. You are allowed to assume that both Ψ and Φ are continuous bounded functions, and therefore you can use the definition of Riemann integral.
2. Prove that any bounded monotonic non-decreasing sequence of real numbers has a limit. In other words, for any sequence $x_1, x_2, \dots, x_n, x_{n+1}, \dots$ such that $\forall i x_i \in \mathbb{R}$ and $\forall i x_{i+1} \geq x_i$ and $\exists M$ such that $\forall i x_i < M$, the limit $\lim_{i \rightarrow \infty} x_i$ exists.
3. Prove that if $X_1, X_2, \dots, X_n, X_{n+1}, \dots$ are random variables then $\lim_n \sup X_n$ is a random variable.
4. Consider the function $f(x)$ defined as $f(x) = 1$ if x is rational and $f(x) = 0$ otherwise. Compute the Lebesgue integral $\int_0^1 f(x)dx$.