

CAP5416

Homework 4

This assignment is due on Friday, April 13, 2007. If you have any difficulty completing the assignment before the deadline, please inform the instructor or the TA promptly.

Problem 1 Simple Questions

1. What are the intrinsic and extrinsic parameters of a stereo system?
2. What is the epipolar constraint and how could one use it to speed up the search for corresponding points?
3. What are the main properties of the essential and fundamental matrices?
4. What kind of 3D reconstruction can be obtained if all the parameters, only the intrinsic parameters, or no parameters can be assumed to be known?

Problem 2 Normlization

We have discussed the Eight-Point algorithm that estimates the fundamental matrix of a stereo system given eight point correspondences. Let $\mathbf{p}_i = (x_i, y_i)$, $\mathbf{q}_i = (u_i, v_i)$ for $1 \leq i \leq 8$ be the eight pairs of corresponding points. Typically, it is not recommended to apply the Eight-Point algorithm directly due to possible numerical problems. For example, the algorithm works with the homogeneous vectors $\hat{\mathbf{p}}_i = (x_i, y_i, 1)$, $\hat{\mathbf{q}}_i = (u_i, v_i, 1)$, and x_i, y_i, u_i, v_i are in pixel coordinates. Therefore, there is a significant difference in magnitude between the first two components and the third component, which is invariably one. This difference in magnitude will cause numerical instability and corrupt the result.

The idea of normalization is to normalize the data before you apply the Eight-Point algorithm. Here is the recipe. Let $\bar{x} = \sum_{i=1}^8 x_i/8$, $\bar{y} = \sum_{i=1}^8 y_i/8$ and

$$\bar{d} = \frac{\sum_{i=1}^8 \sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}}{8}.$$

Then, the normalized data is given as $\hat{\mathbf{p}}_i = [(x_i - \bar{x})/\bar{d}, (y_i - \bar{y})/\bar{d}, 1]^t$, $1 \leq i \leq 8$ (and similarly for $\hat{\mathbf{q}}_i$).

1. Notice that the last component of $\hat{\mathbf{p}}_i$ is always 1 for all i . After the normalization, we can show that the average magnitude of the first two components (considered as a 2×1 vector) of $\hat{\mathbf{p}}_i$ also equals to 1. Demonstrate this.

2. Find a 3×3 matrix H such that $H\mathbf{p}_i = \hat{\mathbf{p}}_i$.
3. Let F be the fundamental matrix estimated from $\hat{\mathbf{p}}_i, \hat{\mathbf{q}}_i$. H and H' are the matrices such that $H\mathbf{p}_i = \hat{\mathbf{p}}_i, H'\mathbf{q}_i = \hat{\mathbf{q}}_i$, respectively. What is the fundamental matrix between \mathbf{p}_i and \mathbf{q}_i ?

Problems 3-4

Problems 13.1 and 13.7 in the textbook.

For these two problems, you don't really need to read the entire chapter. The material presented in this chapter is somewhat old-fashioned. Nothing wrong with it. It is just tedious to sort out all the equations. With this said, I do encourage you to read the chapter, and I will be glad to discuss it individually (or in class).