4a. Space Partition

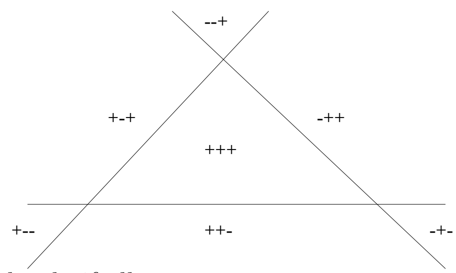
Barycentric coordinates, Convex hull

If $n+1$ points $P_0, \ldots, P_n$ in R^n are in general position (span the space) then they define a unique coordinate system with *barycentric coordinates* $(\alpha_0, \alpha_1, \ldots, \alpha_n), \sum \alpha_i = 1$.

Special barycentric coordinates, the *Cartesian coordinates* arise when one point is made special and called origin, and the difference vectors to the other points are orthogonal. We then drop α_0 since it multiplies the point 0.

In 2D, the *barycentric coordinates of Q with respect to P_1, P_2, P_3* relate to Cartesian coordinates via

$$\begin{bmatrix} P_1^x & P_2^x & P_3^x \\ P_1^y & P_2^y & P_3^y \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} Q^x \\ Q^y \\ 1 \end{bmatrix}$$

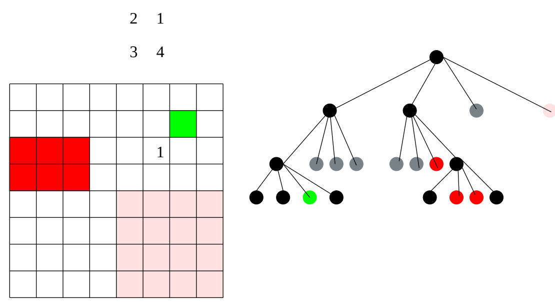


Point Q is strictly inside the triangle if and only if all $\alpha_i > 0$.
Generalization:

$$Q \in \textit{ convex hull } (P_1, \ldots, P_n) : \begin{cases} Q = \sum_{i=1}^n \alpha_i P_i \\ 1 = \sum_{i=1}^n \alpha_i, \alpha_i \geq 0. \end{cases}$$

Collision Detection

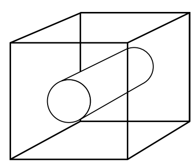
Eulerian (fixed coordinate system) approach. Quad-tree, k-D tree, BSP tree.



Lagrangian (adapted coordinate system): bounding boxes and hierarchical trees.

Constructive Solid Geometry

post-order traversal



$A - B$, $A \cap B$, $A \cup B$, arranged in a tree.