Particles and Shape Grammars

Kinematics

kinematics – motion (trajectories only – without consideration of physical considerations such as forces, friction, inertia etc.) as opposed to *dynamics*.

angle θ, position p

$$\mathbf{p} = F(\theta) \quad \text{(forward) kinematic equation}$$
$$\theta = F^{-1}(\mathbf{p}) \quad \textit{inverse} \text{ kinematic equation}$$

We need the set of angles θ for animation.

Inverse kinematics is diffcult since F^{-1} may not be well-defined (there may be several or no way to achive position $\mathbf{p}$).

Alternative: *keyframe animation* ('inbetweening' or 'interpolation')

Newtonian Particles

position $\mathbf{p}$, velocity $\mathbf{v}$, state $Y := \begin{bmatrix}\mathbf{p}\\ \mathbf{v}\end{bmatrix} \in R^6$ as functions of time t

$$Y' = \begin{bmatrix}\mathbf{p}'\\ \mathbf{v}'\end{bmatrix} = \begin{bmatrix}\mathbf{v}\\ \text{force/mass}\end{bmatrix} =: \mathbf{g}(Y, t)$$

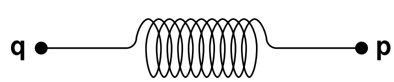
To determine the state at time $t + h$ from the state at time t need to solve the ODE.

Examples of force f: (1) *gravitational force*

$$\mathbf{f} = \begin{bmatrix}0\\ -\gamma\\ 0\end{bmatrix}$$

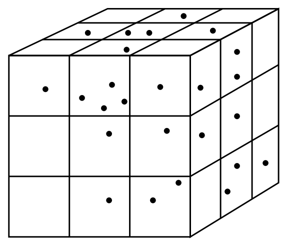
(2) *spring force* (harmonic oscillator) between $\mathbf{p}$ and $\mathbf{q}$.
Define $\mathbf{d} := \mathbf{p} - \mathbf{q}$ and $d := \|\mathbf{d}\|$. (Hooke's law + damping):

$$\mathbf{f} = -\big(k_{\text{spring}}(d - d_0) + k_{\text{damp}}\frac{\mathbf{d}' \cdot \mathbf{d}}{d}\big)\frac{\mathbf{d}}{d}.$$



repulsing force

$$\mathbf{f} = -\frac{k_{\text{repulse}}}{d^2}\frac{1}{d}(\mathbf{p}-\mathbf{q})$$



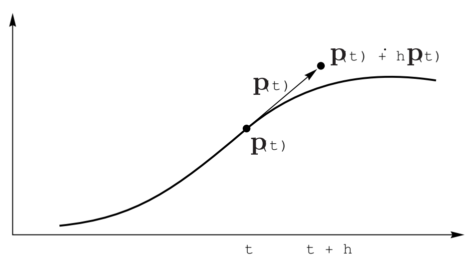
avoid $O(n^2)$

Solving the ODE

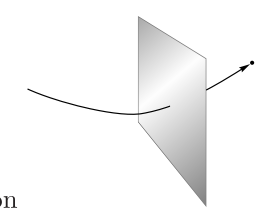
Euler (should not be used! But explains basic forward-solving idea)

$$Y(t+h) = Y(t) + hY'(t) + O(h^2) = Y(t) + hg(Y,t) + O(h^2)$$

stiff ODE's, Runge Kutta solvers, Verlet Integration!
Implicit Euler: $Y(t+h) = Y(t) + h\mathbf{g}(Y(t+h), t+h)$



Constraints



soft (approximate) and *hard* e.g. collision

- Detection — difficult! bounding boxes, quadtrees, binary space partitioning

- Reaction: Let $\bar{\mathbf{p}} := \mathbf{p}(t) - \mathbf{p}(t^*)$ where t is the previous time and t^* the time of intersection, $\mathbf{n}$ is the normal at $\mathbf{p}(t^*)$.
  Then the new point is in the direction

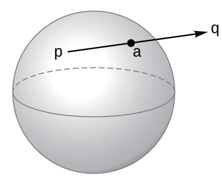
$$\mathbf{p} + 2\big(\bar{\mathbf{p}} - (\bar{\mathbf{p}} \cdot \mathbf{n})\mathbf{n}\big).$$

  This corresponds to an *elastic* collision (no energy loss, deformation).

- Soft constraints with penalty function: $\min \|\mathbf{p} - \mathbf{p}_0\|^2$.

Examples

- `bounce.c`
- sphere bouncing: intercept with sphere of radius r

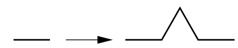


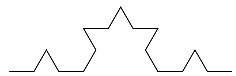
$$\|(1-\lambda)\mathbf{p}(t) + \lambda\mathbf{p}(t+h)\| = r$$

Normal: $\frac{(1-\lambda)\mathbf{p}(t)+\lambda\mathbf{p}(t+h)}{r}$.

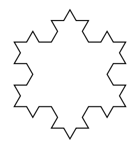
Language based models

tree, shape grammar

Example (Koch curve, snowflake) F → FLFRRFLF 

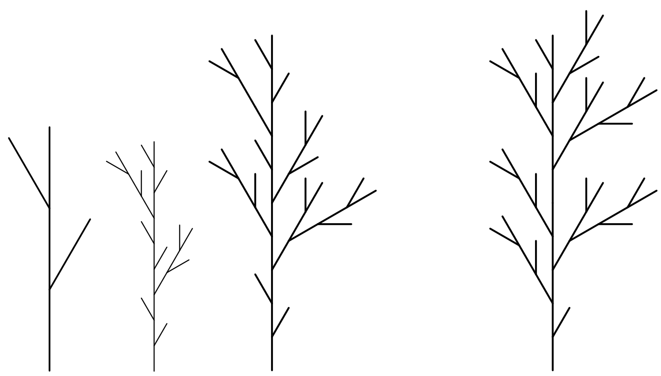


(a)



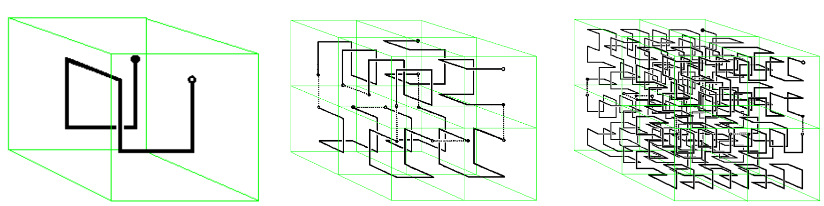
(b)

There exists a unit circle so that the Koch curve inside it has infinite length!



Example F → F[RF]F[LF]F + random
`OPENGL/LRfrac`

Space filling Hilbert curve

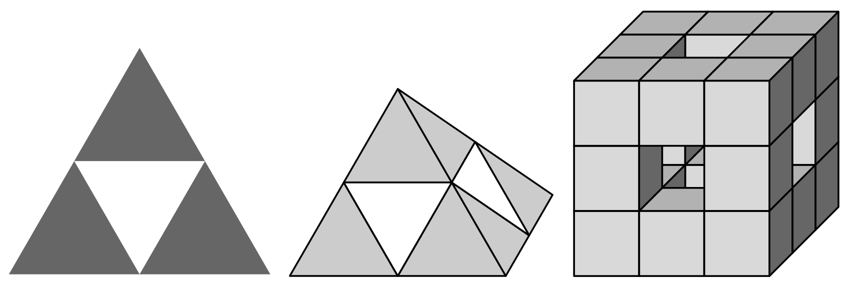


Fractals

self-similar structures

the fractal dimension `dim` : pieces $\cdot$ length$^{\texttt{dim}} = 1$.

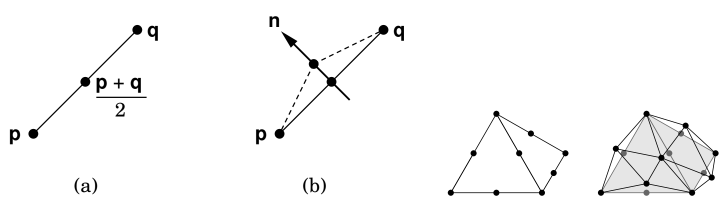
$$\texttt{dim} = \frac{\ln\text{(new) pieces}}{\ln\text{(old edge) length}}$$



$$ln(20)/ln(3) = 2.726833..$$

Fractal mountains

brownian (random) "motion" (displacement) –
underlying Gaussian with variance: (length of old curve)$^{2(2-\texttt{dim})}$ yields fractal dimension `dim`.



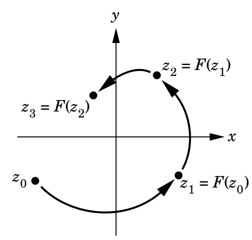
Mandelbrot set

(see also: Julia set)

$z_{k+1} = z_k^2 + c, \qquad z_0 = 0,\ z = x + iy \in C$

`Cex/Angel/mandelbrotcrash`

Mandelbrot set == the set of all c for which this iteration converges.


y
x
$z_2 = F(z_1)$
$z_3 = F(z_2)$
z_0
$z_1 = F(z_0)$