I. Introduction

This documents includes some supplementary results, explanations and tables/images which we could not include in the main paper owing to space restrictions. A list of all the topics covered in this supplement is below:

1) Relationship of NL-SVD bases with the Fourier Bases (Section II)
2) Structurally disparate patches that satisfy the statistical criterion (Section III)
3) Visualizing HOSVD bases (Section IV)
4) Additional Experimental Results on Grayscale Images (with Gaussian noise) (Section V)
5) Effect of Very High Noise on NL-SVD and HOSVD (Section VI)
6) Effect of K (i.e. the number of 'similar patches in the stack) on performance of HOSVD (Section VII)
7) Results on Real-world Degraded Grayscale Images (Section VIII)
8) Additional Experimental Results on Color Images (Section IX)
9) Effect of Random Selection of Patches on HOSVD Results (Section X)
10) Result on a Real-World Color Image (Section XI)

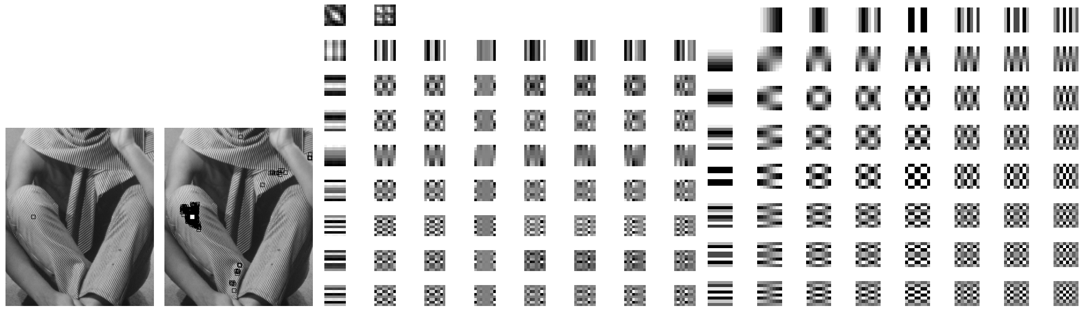
II. Relationship of NL-SVD bases with the Fourier Bases

We now present an example of the NL-SVD bases to show the effect of the structure of the patch and to visualize the corresponding bases. The first example (in Figure 1) is a patch of size 8×8 containing oriented texture from the Barbara image. The patches similar to it (as measured in the noisy version of that image) are shown alongside, as also the learned bases and the DCT bases. (We actually plot 64 outer products of the form $U_i V_j^T$ $(1 \leq i, j \leq 8)$ where U and V are either the learned bases, or the DCT bases respectively). It is well-known that the principal components of natural image patches (in this case, just rows or columns from image patches) are the Fourier bases, as a consequence of the translation invariance property of the covariance between natural images (see section 5.8.2 of the book "Natural Image Statistics: A Probabilistic Approach to Early Computational Vision" by Hyvarinen, Hurri and Hoyer). To see this experimentally, we computed the row-row and column-column covariance matrices of 8×8 patches sampled at every 4 pixels from all the 300 images of the Berkeley database[1] converted to gray-scale (i.e. a total of 2.88×10^6 patches). The eigenvectors of these matrices were very similar to DCT bases (the real parts of the Fourier bases) as measured by the angles between corresponding basis vectors: 0.2, 4, 4, 6.8, 5.6, 6, 4 and 3 degrees. For NL-SVD, the consequence of this result is as follows. If for every reference patch, the correlation matrices were computed from several patches without attention to similarity, we could get a filter very similar to the sliding window DCT filter (modulo asymptotics, and barring the difference due to robust PCA).

III. Structurally disparate patches that satisfy the statistical criterion

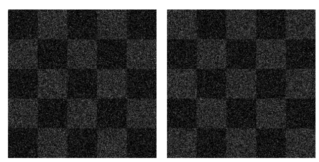
We illustrate an example of a pair of structurally disparate patches that satisfy the statistical criterion for patch similarity described in the manuscript, in Figure 2 which shows two structurally very different images (containing

[1] http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/



(a) (b) (c) (d)

Fig. 1. Barbara image, (a) reference patch, (b) patches similar to the reference patch (similarity measured on noisy image which is not shown here), (c) correlation matrices (top row) and learned bases, (d) DCT bases



(a) (b)

Fig. 2. Motivation for robust PCA: though the patches are structurally different, the difference between the two noisy patches falls below the threshold of $3\sigma^2 n^2$

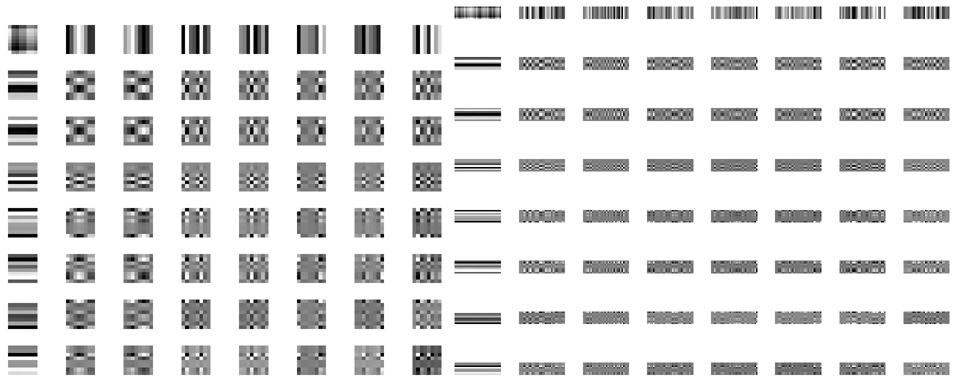
gray-levels of 10 and 40) under the action of $\mathcal{N}(0, 20)$. However their MSE (mean-squared error) = $4075 < \tau_d = 3\sigma^2 = 4800$. To eliminate such 'false positives', we observe that if P_{ref} and P_i are noisy versions of the same patch, the values in $P_{ref} - P_i$ belong to $\mathcal{N}(0, \sqrt{2}\sigma)$. For the two images in Figure 2, we observed that the KS-test yields a p-value very close to 0, thereby providing a better indication of structural dissimilarity.

IV. Visualizing HOSVD bases

In Figure 3, we display the HOSVD bases, i.e. the outer-products of column vectors from the $U^{(1)}$, $U^{(2)}$ and $U^{(3)}$ bases respectively. While the HOSVD bases visually resemble the DCT bases, they are asymmetric unlike the DCT bases.

V. Additional Experimental Results on Grayscale Images

Tables I, II and III gives results on denoising grayscale images that have been corrupted by $\mathcal{N}(0, 15)$, $\mathcal{N}(0, 25)$ and $\mathcal{N}(0, 35)$ respectively.



(a) (b)

Fig. 3. Pictorial view of HOSVD bases - outer products of column vectors from the (a): $U^{(1)}$ and $U^{(2)}$ matrices, and (b): $U^{(2)}$ and $U^{(3)}$ matrices).

VI. Effect of Very High Noise on NL-SVD and HOSVD

In Figures 4, 5 and 6, we show results of the action of NL-SVD and HOSVD on a textured image corrupted with noise from $\mathcal{N}(0,60)$, $\mathcal{N}(0,80)$ and $\mathcal{N}(0,100)$. We note that HOSVD/HOSVD2 is able to reconstruct the texture quite well even at these noise levels, whereas NL-SVD tends to oversmooth them. PSNR results on the Lansel dataset for noise levels $\sigma \in \{60,80,100\}$ are shown in Tables V, VI and VII respectively.

VII. Effect of K (i.e. the number of 'similar patches in the stack) on performance of HOSVD

We have selected $K=30$ as an upper limit to the number of 'similar' patches in the stack, while using HOSVD. We have observed that increasing the value of K does not worsen the denoising results at all. We present these in Tables VIII, IX and X, for all 13 images from the Lansel dataset - at noise levels 10, 20 and 30, where we vary K in the range $\{10,20,30,40,50,60,70\}$. For the barbara image at noise level 20, we also tested for values of K upto and including 130 and noticed similar results. In the rest of the experiments on HOSVD reported in the manuscript, we set $K=30$ as higher values of K did not increase the PSNR by more than 0.02 dB, and increased the computation time greatly (computing HOSVD bases is $O(K^3)$). Very small values of K ($K<10$) lead to lower PSNR values as the self-similarity between patches is not adequately exploited. The point to note is that even if K is large, the stack of patches is created using *only those patches that are similar to the reference patch*. This similarity is dictated by the noise model (i.e. mean squared distance between the candidate patch and the reference patch must be less than $3\sigma^2$) as discussed in the manuscript. Thus, we conclude that the choice of K does not take away from the principled nature of our algorithms (or its implementation), as using higher values of K does not deteriorate the quality of the denoised output.

TABLE I

PSNR VALUES FOR NOISE LEVEL $\sigma = 15$ ON THE BENCHMARK DATASET

Image #	NL-SVD	NL-Means	KSVD	HOSVD	HOSVD2	3DDCT	BM3D1	BM3D2	Oracle	LPG-PCA1	LPG-PCA2
13	33.262	32.223	33.597	33.341	33.742	32.825	33.506	33.949	39.502	33.077	33.652
12	32.283	31.363	32.375	32.836	33.182	32.008	32.587	33.057	38.766	32.496	32.960
11	31.454	30.566	31.706	31.704	31.933	30.899	31.710	32.039	36.910	31.534	31.638
10	31.320	30.342	31.394	31.576	31.847	30.963	31.725	32.049	37.637	31.358	31.494
9	31.854	31.355	32.271	32.201	32.096	31.603	32.095	32.132	35.364	32.234	32.297
8	29.537	29.159	30.051	29.798	30.099	29.229	29.904	30.262	37.562	30.017	30.284
7	31.296	30.617	31.508	31.651	31.830	30.703	31.604	31.865	36.835	31.45	31.505
6	33.487	32.166	33.712	33.688	33.970	33.222	33.737	34.133	38.506	33.455	33.966
5	30.388	29.835	30.482	30.568	30.889	29.759	30.700	30.973	37.013	30.810	30.878
4	27.557	27.461	27.969	27.101	27.947	27.305	27.881	28.166	35.793	28.206	28.103
3	33.138	32.107	33.199	33.326	33.406	32.852	33.375	33.594	37.534	33.101	33.450
2	28.129	28.080	28.564	27.761	28.416	27.619	28.405	28.718	35.905	28.585	28.504
1	34.628	32.817	34.748	34.848	35.017	34.375	34.771	35.270	39.248	34.411	35.115
13	0.905	0.855	0.910	0.898	0.902	0.910	0.901	0.916	0.967	0.871	0.913
12	0.910	0.876	0.909	0.916	0.909	0.921	0.913	0.923	0.969	0.901	0.920
11	0.836	0.810	0.841	0.849	0.822	0.852	0.845	0.853	0.935	0.838	0.843
10	0.853	0.821	0.852	0.866	0.844	0.870	0.864	0.874	0.955	0.852	0.858
9	0.771	0.773	0.789	0.791	0.758	0.780	0.782	0.776	0.886	0.799	0.794
8	0.940	0.934	0.946	0.943	0.931	0.948	0.944	0.949	0.990	0.946	0.948
7	0.820	0.799	0.824	0.838	0.799	0.841	0.832	0.841	0.942	0.826	0.829
6	0.881	0.837	0.884	0.883	0.880	0.889	0.883	0.893	0.949	0.866	0.892
5	0.828	0.799	0.823	0.838	0.804	0.844	0.834	0.842	0.950	0.843	0.851
4	0.823	0.826	0.835	0.814	0.793	0.838	0.831	0.842	0.956	0.844	0.834
3	0.855	0.822	0.855	0.857	0.852	0.856	0.857	0.861	0.929	0.844	0.861
2	0.825	0.818	0.837	0.814	0.785	0.837	0.831	0.845	0.964	0.839	0.834
1	0.887	0.836	0.888	0.888	0.888	0.889	0.887	0.897	0.952	0.869	0.897

VIII. RESULTS ON REAL-WORLD DEGRADED GRAYSCALE IMAGES

We show two results on real-world degraded images captured under poor lighting conditions, where the noise is non-Gaussian (most likely Poisson). We make the simplifying assumption that the noise model is Gaussian and run the experiments on the noisy image with various σ values, picking the visually most pleasing result. The original and denoised images are shown in Figure 7, where for the two images, we used noise models $\mathcal{N}(0, 30)$ and $\mathcal{N}(0, 7)$ respectively.

IX. EXPERIMENTAL RESULTS ON COLOR IMAGES

Tables XI and XII give results on denoising color images that have been corrupted by noise from $\mathcal{N}(0, 10)$ and $\mathcal{N}(0, 40)$ respectively. Figure 8 shows two original images, their noisy versions under $\mathcal{N}(0, 30)$, and their denoised versions using both 3D-IHOSVD, BM3D2 and 4DHOSVD(2). Note that 4DHOSVD is devoid of any color artifacts

TABLE II

PSNR VALUES FOR NOISE LEVEL $\sigma = 25$ ON THE BENCHMARK DATASET

Image #	NL-SVD	NL-Means	KSVD	HOSVD	HOSVD2	3DDCT	BM3D1	BM3D2	Oracle	LPG-PCA1	LPG-PCA2
13	30.845	28.835	31.145	31.038	31.173	30.324	30.955	31.512	36.212	28.890	30.124
12	29.766	28.135	29.552	30.439	30.678	29.367	29.870	30.595	35.030	28.442	29.060
11	29.207	27.495	29.188	29.482	29.640	28.476	29.306	29.782	34.292	27.906	28.437
10	28.736	26.916	28.689	29.297	29.423	28.215	29.120	29.639	34.794	27.460	27.852
9	30.484	28.906	30.629	30.698	30.556	30.287	30.540	30.880	33.389	29.151	30.086
8	26.834	25.764	27.225	27.422	27.682	26.218	27.262	27.719	33.669	26.019	26.102
7	29.094	27.608	29.152	29.588	29.673	28.690	29.433	29.834	34.074	27.928	28.433
6	31.329	29.202	31.279	31.470	31.592	30.852	31.294	31.957	35.678	29.300	30.487
5	27.763	26.367	27.617	28.200	28.332	27.136	28.061	28.340	33.344	27.267	27.554
4	24.929	24.488	25.204	25.019	25.285	24.186	25.097	25.435	31.635	24.093	24.099
3	30.993	28.802	30.801	31.349	31.281	30.785	31.170	31.626	35.239	29.492	30.763
2	25.792	24.894	25.900	25.823	26.057	24.741	25.876	26.106	31.623	25.028	25.011
1	32.456	29.584	32.367	32.685	32.662	32.127	32.343	33.120	36.856	30.081	31.685
13	0.868	0.760	0.876	0.854	0.871	0.870	0.852	0.885	0.951	0.690	0.852
12	0.850	0.780	0.849	0.870	0.880	0.860	0.857	0.884	0.943	0.775	0.848
11	0.771	0.700	0.769	0.784	0.793	0.758	0.777	0.799	0.909	0.689	0.752
10	0.777	0.697	0.773	0.800	0.808	0.769	0.792	0.816	0.935	0.700	0.751
9	0.729	0.687	0.733	0.737	0.730	0.727	0.732	0.739	0.841	0.669	0.730
8	0.886	0.864	0.896	0.906	0.913	0.874	0.902	0.913	0.977	0.869	0.870
7	0.740	0.681	0.739	0.766	0.774	0.732	0.759	0.778	0.910	0.673	0.723
6	0.840	0.738	0.842	0.834	0.844	0.842	0.828	0.858	0.928	0.706	0.830
5	0.730	0.664	0.723	0.749	0.757	0.711	0.741	0.756	0.911	0.691	0.740
4	0.713	0.690	0.725	0.720	0.743	0.652	0.723	0.745	0.915	0.666	0.669
3	0.815	0.725	0.815	0.814	0.816	0.821	0.808	0.828	0.909	0.702	0.817
2	0.707	0.667	0.713	0.715	0.736	0.636	0.719	0.735	0.919	0.669	0.668
1	0.849	0.735	0.850	0.844	0.846	0.854	0.836	0.864	0.936	0.710	0.841

that can be noticed in the BM3D2 output that may have arisen due to the treatment of the three decorrelated color channels as being independent.

X. Effect of Random Selection of Patches on HOSVD Results

In this section, we show the effect of random selection of patches to create the patch stack (as opposed to building the patch stack from similar patches). Essentially, we observe that the denoised output in the former case is both grainy and blurry. Though we show a sample result on a color image, the issue is equally applicable for grayscale images. Figure 9 illustrates this.

XI. Result on a Real-World Color Image

In figure 10, we show a denoising output on a real-world color image.

TABLE III

PSNR AND SSIM VALUES FOR NOISE LEVEL $\sigma = 35$ ON THE BENCHMARK DATASET

Image #	NL-SVD	NL-Means	HOSVD	3DDCT	BM3D1	BM3D2	Oracle
13	28.905	26.542	29.322	28.625	29.259	29.897	33.834
12	27.497	25.663	28.547	27.559	27.944	28.914	32.188
11	27.419	25.330	27.903	26.869	27.664	28.287	32.523
10	26.931	24.750	27.572	26.522	27.342	28.010	32.796
9	29.436	26.895	29.533	29.247	29.313	29.913	32.197
8	24.925	23.628	25.822	24.701	25.516	26.065	31.128
7	27.692	25.732	28.116	27.354	27.930	28.440	32.378
6	29.466	27.057	29.847	29.386	29.676	30.545	33.778
5	25.948	24.237	26.477	25.531	26.312	26.590	31.071
4	23.385	22.565	23.537	22.394	23.408	23.804	29.055
3	29.101	26.321	29.469	28.992	29.302	29.900	33.358
2	24.167	23.078	24.575	23.504	24.466	24.730	29.155
1	30.824	27.275	30.822	30.462	30.450	31.349	34.869
13	0.838	0.674	0.810	0.843	0.814	0.860	0.937
12	0.790	0.681	0.815	0.813	0.801	0.846	0.912
11	0.716	0.600	0.727	0.709	0.718	0.755	0.892
10	0.711	0.588	0.736	0.710	0.724	0.766	0.918
9	0.702	0.602	0.698	0.705	0.691	0.715	0.821
8	0.831	0.793	0.869	0.829	0.859	0.877	0.960
7	0.687	0.588	0.707	0.684	0.698	0.728	0.888
6	0.798	0.640	0.783	0.813	0.779	0.830	0.909
5	0.655	0.564	0.678	0.647	0.669	0.693	0.875
4	0.611	0.569	0.632	0.538	0.630	0.657	0.875
3	0.778	0.631	0.761	0.789	0.759	0.797	0.892
2	0.602	0.564	0.642	0.559	0.637	0.654	0.874
1	0.814	0.632	0.790	0.824	0.782	0.830	0.922

TABLE IV

PSNR VALUES AT HIGH NOISE

Image	σ	NL-SVD	HOSVD	HOSDV2	BM3D1	BM3D2
texture1	60	21.83	22.44	22.58	22.24	22.79
texture2	60	20.21	21.10	21.33	20.84	21.59
texture1	80	20.61	21.25	21.37	21.15	21.83
texture2	80	18.66	19.61	19.80	19.5	20.3
texture1	100	19.51	20.22	20.32	20.29	21.11
texture2	100	17.92	18.54	18.65	18.64	19.2

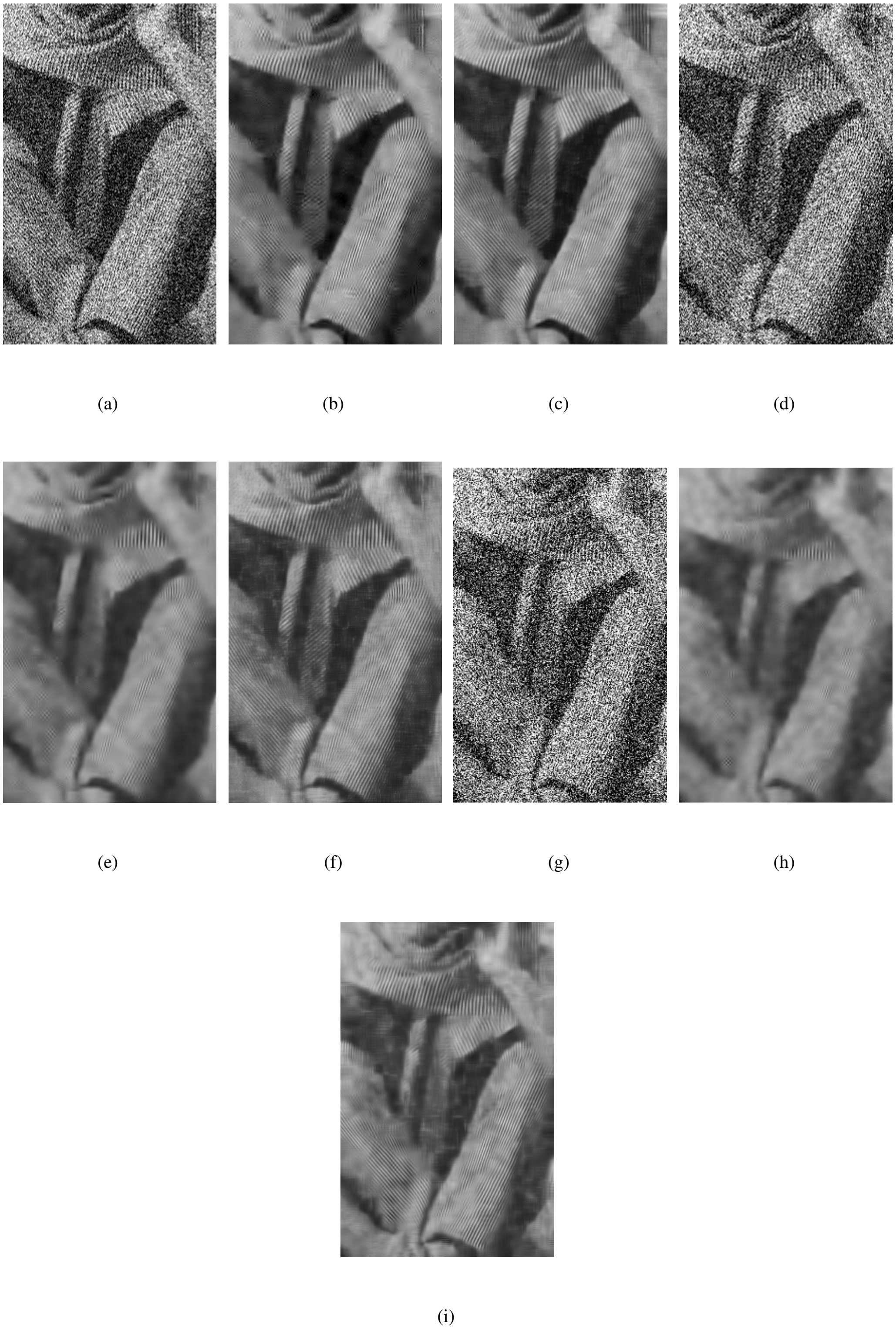


Fig. 4. Corrupted images under (a) $\mathcal{N}(0, 60)$, (d) $\mathcal{N}(0, 80)$ and (g) $\mathcal{N}(0, 100)$. Corresponding output of NL-SVD: (b), (e) and (h). Corresponding output of HOSVD: (c), (f) and (i). Please zoom in for better view.

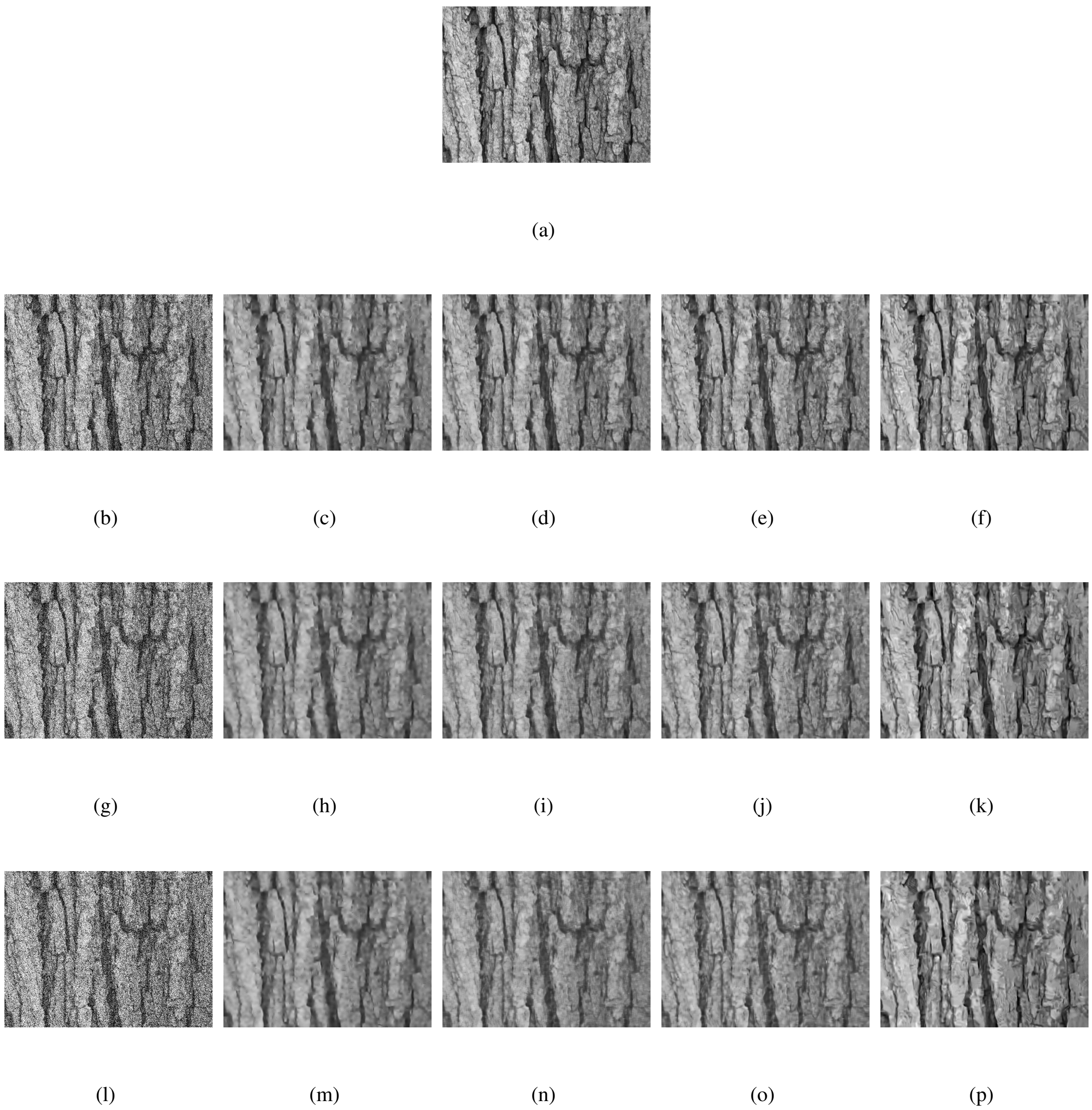


Fig. 5. (a) Texture image; For $\mathcal{N}(0, 60)$, (b) Noisy version; Reconstructions using (c) NL-SVD, (d) HOSVD, (e) HOSVD2, (f) BM3D2; For $\mathcal{N}(0, 80)$, (g) Noisy version; Reconstructions using (h) NL-SVD, (i) HOSVD, (j) HOSVD2, (k) BM3D2; For $\mathcal{N}(0, 100)$, (l) Noisy version; Reconstructions using (m) NL-SVD, (n) HOSVD, (o) HOSVD2, (p) BM3D2. PSNR values in Table IV.

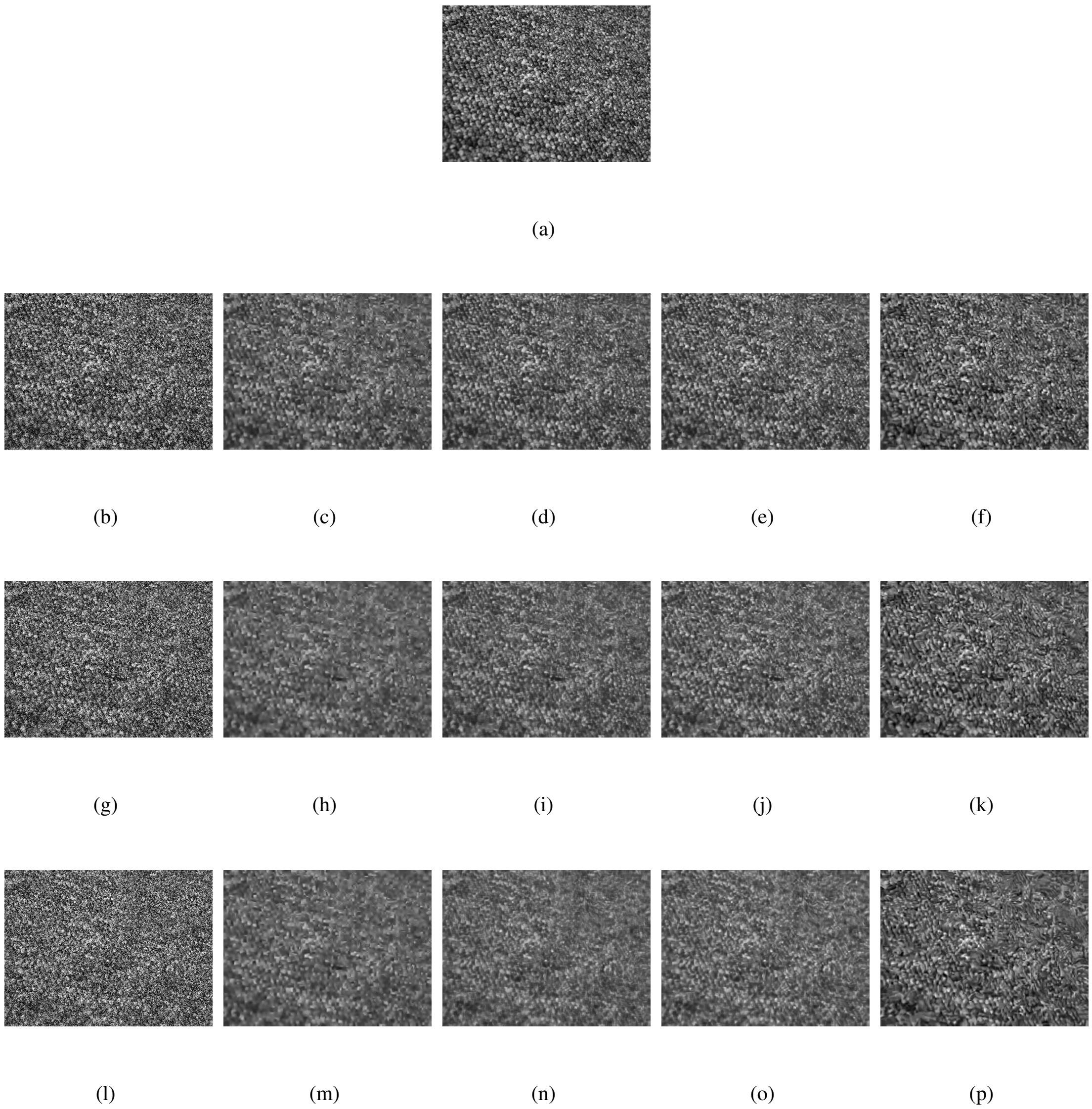


Fig. 6. (a) Texture image; For $\mathcal{N}(0, 60)$, (b) Noisy version; Reconstructions using (c) NL-SVD, (d) HOSVD, (e) HOSVD2, (f) BM3D2; For $\mathcal{N}(0, 80)$, (g) Noisy version; Reconstructions using (h) NL-SVD, (i) HOSVD, (j) HOSVD2, (k) BM3D2; For $\mathcal{N}(0, 100)$, (l) Noisy version; Reconstructions using (m) NL-SVD, (n) HOSVD, (o) HOSVD2, (p) BM3D2. PSNR values in Table IV.

TABLE V

PSNR VALUES AT $\sigma = 60$

Image #	HOSVD	HOSVD2	BM3D1	BM3D2
13	25.728, 0.784	26.054, 0.799	25.703, 0.719	25.792, 0.767
12	24.942, 0.709	25.527, 0.739	25.059, 0.688	25.495, 0.729
11	24.908, 0.628	25.286, 0.652	24.959, 0.607	25.136, 0.639
10	24.659, 0.619	25.112, 0.654	24.671, 0.599	24.832, 0.631
9	27.172, 0.653	27.653, 0.673	27.007, 0.617	27.176, 0.641
8	22.740, 0.753	23.241, 0.781	23.011, 0.774	23.266, 0.788
7	25.510, 0.609	25.857, 0.634	25.590, 0.598	25.723, 0.624
6	27.085, 0.740	27.571, 0.769	26.791, 0.673	27.005, 0.719
5	23.019, 0.541	23.291, 0.565	23.577, 0.550	23.493, 0.569
4	21.241, 0.446	21.332, 0.457	21.538, 0.481	21.718, 0.507
3	25.886, 0.717	26.259, 0.737	26.098, 0.660	26.094, 0.694
2	22.202, 0.476	22.380, 0.491	22.541, 0.515	22.616, 0.527
1	27.463, 0.745	27.867, 0.773	27.519, 0.680	27.576, 0.724

TABLE VI

PSNR VALUES AT $\sigma = 80$

Image #	HOSVD	HOSVD2	BM3D1	BM3D2
13	23.014, 0.742	23.320, 0.757	23.296, 0.655	23.295, 0.718
12	22.717, 0.633	23.290, 0.664	22.988, 0.596	23.412, 0.652
11	23.168, 0.571	23.547, 0.596	23.305, 0.537	23.432, 0.579
10	23.039, 0.553	23.487, 0.587	23.168, 0.526	23.280, 0.565
9	25.286, 0.626	25.759, 0.645	25.124, 0.566	25.311, 0.608
8	20.700, 0.640	21.435, 0.693	21.207, 0.692	21.534, 0.715
7	23.802, 0.560	24.133, 0.584	24.065, 0.537	24.105, 0.573
6	24.947, 0.706	25.309, 0.730	24.753, 0.608	24.945, 0.670
5	21.129, 0.483	21.395, 0.505	21.703, 0.476	21.578, 0.503
4	20.130, 0.348	20.241, 0.357	20.583, 0.402	20.646, 0.416
3	23.451, 0.678	23.811, 0.696	23.744, 0.594	23.704, 0.645
2	20.926, 0.407	21.176, 0.427	21.314, 0.444	21.333, 0.454
1	25.073, 0.713	25.435, 0.739	25.325, 0.612	25.311, 0.677

TABLE VII

PSNR VALUES AT $\sigma = 100$

Image #	HOSVD	HOSVD2	BM3D1	BM3D2
13	20.873, 0.713	21.140, 0.725	21.290, 0.610	21.197, 0.680
12	21.023, 0.570	21.446, 0.593	21.288, 0.522	21.526, 0.581
11	21.745, 0.531	22.085, 0.551	21.992, 0.488	22.045, 0.532
10	21.701, 0.505	22.059, 0.532	22.054, 0.479	22.118, 0.521
9	23.606, 0.607	23.982, 0.622	23.616, 0.534	23.737, 0.584
8	19.004, 0.524	20.000, 0.614	19.630, 0.597	20.020, 0.634
7	22.214, 0.526	22.482, 0.545	22.619, 0.492	22.548, 0.533
6	23.104, 0.675	23.478, 0.695	23.297, 0.567	23.326, 0.634
5	19.602, 0.441	19.876, 0.464	20.174, 0.425	19.970, 0.457
4	19.461, 0.308	19.574, 0.317	19.877, 0.351	19.846, 0.359
3	21.473, 0.649	21.792, 0.662	21.835, 0.543	21.679, 0.600
2	19.751, 0.362	20.032, 0.382	20.298, 0.400	20.256, 0.413
1	23.147, 0.691	23.415, 0.711	23.585, 0.565	23.449, 0.645

TABLE VIII

EFFECT OF K ON PERFORMANCE OF HOSVD AT $\sigma = 10$

Image #	10	20	30	40	50	60	70
13	35.46	35.55	35.58	35.6	35.61	35.62	35.62
12	34.69	34.72	34.74	34.76	34.77	34.77	34.77
11	33.6	33.57	33.59	33.61	33.64	33.66	33.68
10	33.71	33.70	33.71	33.72	33.74	33.75	33.76
9	33.31	33.29	33.33	33.39	33.45	33.5	33.55
8	32.10	32.11	32.11	32.11	32.11	32.11	32.11
7	33.29	33.26	33.28	33.31	33.34	33.36	33.38
6	35.54	35.6	35.63	35.66	35.67	35.68	35.69
5	32.85	32.82	32.83	32.84	32.86	32.87	32.87
4	30.13	30.11	30.11	30.12	30.13	30.13	30.14
3	34.90	34.90	34.91	34.93	34.94	34.96	34.97
2	30.52	30.51	30.51	30.52	30.53	30.53	30.54
1	36.57	36.66	36.71	36.72	36.72	36.71	36.71

TABLE IX

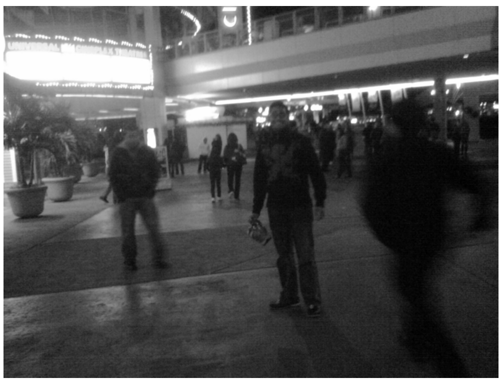
EFFECT OF K ON PERFORMANCE OF HOSVD AT $\sigma = 20$

Image #	10	20	30	40	50	60	70
13	31.81	32.09	32.18	32.23	32.26	32.27	32.26
12	31.29	31.47	31.55	31.59	31.61	31.62	31.61
11	30.38	30.45	30.49	30.52	30.54	30.56	30.58
10	30.28	30.33	30.35	30.37	30.38	30.39	30.39
9	31.18	31.33	31.28	31.42	31.44	31.45	31.46
8	28.38	28.43	28,51	28.58	28.64	28.68	28.72
7	30.38	30.44	30.47	30.51	30.53	30.56	30.58
6	32.31	32.49	32.58	32.62	32.64	32.65	32.65
5	29.17	29.18	29.2	29.23	29.27	29.29	29.30
4	26.09	26.06	26.07	26.08	26.11	26.13	26.14
3	32.00	32.23	32.39	32.3	32.31	32.3	32.28
2	26.81	26.76	26.78	26.8	26.84	26.87	26.89
1	33.2	33.56	33.65	33.69	33.71	33.70	33.67

TABLE X

EFFECT OF K ON PERFORMANCE OF HOSVD AT $\sigma = 30$

Image #	10	20	30	40	50	60	70
13	29.61	30	30.13	30.20	30.24	30.26	30.26
12	28.86	29.17	29.27	29.34	29.38	29.38	39.38
11	28.34	28.54	28.59	28.62	28.65	28.66	28.66
10	28.02	28.15	28.2	28.23	28.24	28.26	28.26
9	29.57	29.96	30.06	30.09	30.11	30.10	30.08
8	26.43	26.53	26.61	26.70	26.77	26.83	26.87
7	28.53	28.71	28.76	28.79	28.82	28.83	28.84
6	29.84	30.28	30.41	30.48	30.51	30.52	30.50
5	26.99	27.06	27.10	27.13	27.16	27.18	27.2
4	24.13	24.14	24.14	24.15	24.19	24.23	24.27
3	29.65	30.08	30.19	30.22	30.22	30.21	30.17
2	25.14	25.07	25.10	25.14	25.19	25.24	25.28
1	30.68	31.25	31.40	31.47	31.48	31.48	31.48



(a)

(b)

(c)

(d)

Fig. 7. (a) and (b): Real-world corrupted images, (c) and (d): their respective denoised versions, assuming $\sigma = 30$ and $\sigma = 7$ respectively. Zoom into pdf file for a detailed view.


(a)


(b)


(c)

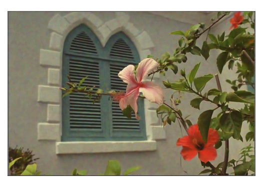

(d)


(e)


(f)


(g)


(h)


(i)


(j)


(k)


(l)

Fig. 8. Two images from Kodak database (# 18,7): from left to right, original, noisy ($\sigma = 30$ on R, G, B) and denoised with 3D-IHOSVD, BM3D2, 4DHOSVD and 4DHOSVD2. Zoom into pdf file for a detailed view. PSNR values in the main paper, images 18, 24.

TABLE XI

PSNR, SSIM results for color images corrupted by $\mathcal{N}(0, 10)$ for BM3D1, BM3D2, NLMeans, 3D-IHOSVD and 4DHOSVD

Image #	BM3D1	BM3D2	NLM	3D-IHOSVD	4DHOSVD
1	34.651, 0.955	34.651, 0.955	31.386, 0.941	33.823, 0.950	33.910, 0.945
2	36.452, 0.922	36.452, 0.922	34.586, 0.909	35.115, 0.906	35.531, 0.898
3	38.848, 0.961	38.848, 0.961	35.991, 0.937	37.637, 0.956	38.293, 0.954
4	36.812, 0.935	36.812, 0.935	34.431, 0.917	35.915, 0.926	35.972, 0.916
5	35.081, 0.965	35.081, 0.965	31.973, 0.952	34.392, 0.962	34.377, 0.959
6	35.149, 0.945	35.466, 0.950	32.487, 0.935	34.768, 0.945	34.889, 0.939
7	37.994, 0.967	38.457, 0.971	35.205, 0.952	37.426, 0.968	38.021, 0.969
8	34.652, 0.956	34.812, 0.959	31.934, 0.951	34.139, 0.954	34.513, 0.950
9	37.751, 0.944	38.141, 0.947	35.396, 0.933	37.415, 0.945	37.858, 0.942
10	37.559, 0.944	37.959, 0.947	35.248, 0.928	37.224, 0.945	37.664, 0.941
11	35.605, 0.934	36.039, 0.941	33.374, 0.922	35.378, 0.934	35.230, 0.925
12	37.149, 0.931	37.612, 0.937	35.210, 0.917	36.745, 0.932	37.095, 0.924
13	33.051, 0.946	33.373, 0.953	30.281, 0.942	32.650, 0.947	32.591, 0.943
14	34.650, 0.934	35.059, 0.941	32.453, 0.926	34.049, 0.933	34.281, 0.925
15	36.534, 0.938	36.916, 0.943	34.690, 0.928	36.201, 0.938	36.599, 0.929
16	36.474, 0.940	36.977, 0.947	34.240, 0.925	36.166, 0.941	36.278, 0.935
17	36.923, 0.943	37.310, 0.947	34.758, 0.931	36.557, 0.943	36.711, 0.938
18	34.461, 0.934	34.850, 0.938	32.084, 0.926	33.938, 0.934	33.963, 0.928
19	35.882, 0.938	36.311, 0.944	33.776, 0.927	35.524, 0.938	35.450, 0.931
20	35.644, 0.959	35.729, 0.963	34.284, 0.953	35.081, 0.958	37.063, 0.954
21	35.685, 0.947	35.969, 0.951	32.891, 0.936	35.387, 0.948	35.398, 0.944
22	35.030, 0.927	35.497, 0.934	33.154, 0.916	34.787, 0.927	34.797, 0.921
23	38.186, 0.951	38.657, 0.955	35.927, 0.934	37.757, 0.952	38.617, 0.951
24	34.827, 0.953	35.157, 0.958	32.731, 0.942	34.566, 0.954	34.916, 0.951

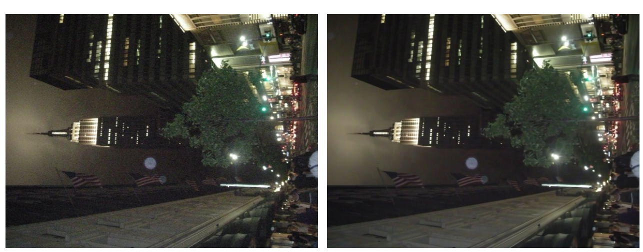
(a) (b) (c) (d)

Fig. 9. (a) A clean image and (b) its corrupted version (under $\mathcal{N}(0, 20)$) [PSNR 22]. 4DHOSVD denoised outputs with (c) random selection of patches [PSNR 30.23], and (d) similarity-based selection of patches in the patch stack [PSNR 32.14]. Zoom into pdf file for a better view.

TABLE XII

PSNR RESULTS FOR COLOR IMAGES (AND SSIM VALUES ON *gray-scale versions*) CORRUPTED BY $\mathcal{N}(0, 40)$

Image #	BM3D1	BM3D2	NLM	3D-IHOSVD	4DHOSVD	4DHOSVD2
1	26.314, 0.739	26.478, 0.747	24.400, 0.682	26.143, 0.742	26.172, 0.712	26.632, 0.740
2	29.197, 0.766	29.457, 0.777	27.206, 0.670	29.043, 0.761	30.153, 0.771	30.391, 0.778
3	30.541, 0.834	30.892, 0.864	27.468, 0.687	29.947, 0.827	31.481, 0.848	31.835, 0.857
4	29.567, 0.773	29.818, 0.791	26.927, 0.656	29.227, 0.774	30.091, 0.777	30.453, 0.790
5	25.946, 0.774	26.258, 0.788	24.181, 0.697	26.009, 0.787	26.346, 0.776	27.094, 0.812
6	27.034, 0.764	27.165, 0.777	25.074, 0.680	26.816, 0.769	27.511, 0.748	28.036, 0.774
7	29.667, 0.860	30.234, 0.891	26.206, 0.696	29.246, 0.859	30.364, 0.882	30.938, 0.892
8	26.256, 0.828	26.339, 0.832	24.067, 0.753	26.248, 0.838	26.970, 0.829	27.550, 0.844
9	30.644, 0.834	31.190, 0.868	27.071, 0.678	30.056, 0.830	31.117, 0.853	31.599, 0.865
10	30.133, 0.812	30.661, 0.843	26.778, 0.659	29.726, 0.815	30.807, 0.831	31.310, 0.843
11	27.822, 0.745	28.090, 0.759	25.739, 0.652	27.732, 0.754	28.262, 0.750	28.784, 0.771
12	30.219, 0.789	30.581, 0.816	27.349, 0.666	29.642, 0.792	31.363, 0.807	31.579, 0.815
13	24.227, 0.662	24.572, 0.668	23.368, 0.653	24.196, 0.672	24.272, 0.635	24.830, 0.675
14	26.875, 0.726	27.133, 0.735	25.184, 0.664	26.808, 0.734	27.187, 0.715	27.675, 0.741
15	28.518, 0.804	28.499, 0.815	26.714, 0.720	28.175, 0.801	30.676, 0.819	30.990, 0.829
16	29.230, 0.753	29.599, 0.780	26.511, 0.628	28.938, 0.764	29.552, 0.762	29.972, 0.782
17	28.815, 0.800	28.968, 0.814	26.386, 0.692	28.610, 0.800	29.941, 0.817	30.440, 0.831
18	26.273, 0.716	26.637, 0.733	24.939, 0.641	25.377, 0.674	26.451, 0.710	27.098, 0.747
19	28.926, 0.770	29.143, 0.789	26.339, 0.668	28.386, 0.767	29.459, 0.784	29.771, 0.794
20	26.257, 0.861	25.871, 0.863	25.059, 0.788	25.699, 0.840	30.943, 0.856	31.240, 0.861
21	27.695, 0.796	28.093, 0.827	25.558, 0.675	26.616, 0.774	27.895, 0.810	28.402, 0.829
22	28.046, 0.726	28.394, 0.744	26.065, 0.632	27.456, 0.704	28.368, 0.725	28.754, 0.745
23	30.653, 0.860	31.242, 0.890	27.814, 0.710	30.113, 0.865	32.343, 0.884	32.645, 0.888
24	26.381, 0.769	26.639, 0.784	24.856, 0.676	25.178, 0.730	26.966, 0.762	27.611, 0.788



(a) (b)

Fig. 10. (a: Noisy image taken from digital camera, (b): Denoised version using HOSVD, assuming camera noise was from $\mathcal{N}(0, 6)$. Zoom into pdf file for a detailed view.