COT5405: ANALYSIS OF ALGORITHMS

Homework # 5

Due date: Apr 27, 2006, Thursday (noon)

Your solutions should be concise, but complete, and typed or handwritten clearly. Feel free to consult textbooks, journal and conference papers and also each other, but write the solutions yourself and cite your sources. Answer **only five** of the following six questions. Each problem is worth 20 pts.

1. Give a linear-time algorithm to determine whether or not a string T is a cyclic rotation of another string T'. For example, car and arc are cyclic rotations of each other.
2. Prove that if **NP** $\neq$ **co-NP** then **P** $\neq$ **NP**.
3. PARTITION is the problem of deciding, given a set S of numbers, whether there exist a subset A of S whose sum equals the sum of the complement of A, i.e., $\sum_{x \in A} x = \sum_{y \in S-A} y$.

   SUBSET-SUM is the problem of deciding, given a set S of numbers and a target sum t, whether there exist a subset A whose sum equals the target, i.e., $\sum_{x \in A} x = t$.

   (a) Give a polynomial-time reduction from PARTITION to SUBSET-SUM.

   (b) Give a polynomial-time reduction from SUBSET-SUM to PARTITION.
4. Consider the class of satisfiable boolean formulas in conjunctive normal form (CNF) in which each clause contains two literals, 2-SAT $= \{\varphi \in \text{SAT} \mid \varphi \text{ is 2-CNF}\}$.

   (a) Is 2-SAT in **NP**?

   (b) Is there a polynomial-time algorithm for deciding whether or not a boolean formula in 2-CNF is satisfiable? If yes, describe and analyze your algorithm. Otherwise, show that 2-SAT $\in$ **NPC**.
5. Professor Karagöz proposes the following heuristic to solve the vertex-cover problem. Repeatedly select a vertex of highest degree, and remove all of its incident edges. Give an example to show that the professor's heuristic does not have an approximation ratio of 2.
6. BIN PACKING Problem: Given a set of n objects, where the size s_i of the ith object satisfies $0 < s_i < 1$, we wish to pack all the objects into the minimum number of unit-size bins. Each bin can hold any subset of the objects whose total size does not exceed 1.

   Let $S = \sum_{i=1}^{n} s_i$. Also, in part (c) to (e) of this problem, consider the FIRST-FIT Heuristic which takes each object in turn and places it into the first bin that can accomdate it.

   (a) Suppose you already know that the SUBSET-SUM problem is NP-hard. Prove that the BIN PACKING problem is also NP-Hard.

   (b) Argue that the optimal number of bins required is at least $\lceil S \rceil$.

   (c) Argue that the FIRST-FIT heuristic leaves at most one bin less than half full.

   (d) Prove that the number of bins used by the FIRST-FIT heuristic is never more than $2\lceil S \rceil$.

   (e) Prove an approximation ratio of 2 for the FIRST-FIT heuristic.

   (f) Describe an efficient implementation of the FIRST-FIT heuristic and analyze its running time.