COT5405: ANALYSIS OF ALGORITHMS

Homework # 4

Due date: Apr 4, 2006, Tuesday (beginning of the class)

Your solutions should be concise, but complete, and typed or handwritten clearly. Feel free to consult textbooks, journal and conference papers and also each other, but write the solutions yourself and cite your sources. Answer **only five** of the following six questions. Each problem is worth 20 pts.

1. Describe an $O(n^2 \log n)$-time algorithm to determine whether any three points of a set of n points are collinear. Assume two dimensions and exact arithmetic (no round-off errors).

2. Implement at least two of the convex hull algorithms discussed in class using your favorite programming language. Test your algorithms on data sets of various size (e.g., 100-10K) and of various distribution (e.g, random, convex position). Plot the computation time with respect to input size for each input distribution.

3. A *simple polygon* is a region enclosed by a single chain of edges that does not intersect itself. Recall Graham's scan algorithm discussed in class. First, we compute a star-shaped simple polygon of the point set. Then we used a walking scheme with three markers to compute the convex hull of the star-shaped simple polygon.

   (a) Give a simple polygon for which this second stage fails to produce the convex hull.

   (b) Describe an algorithm to construct the convex hull of any simple polygon in linear time.

4. Recall Chan's algorithm described in class. Given a set P of n points we first partition P into n/h subsets each of size h, where h is the size of $\mathit{ConvexHull}(P)$. Then, we compute the convex hull of each subset in $O(h \log h)$ time. Finally we use a Jarvis-type wrapping scheme which employs $O(\log h)$ time queries for each subset. The complexity of this wrapping step is $O(n \log h)$. Describe an alternative wrapping scheme that takes only $O(n)$ time.

5. In the *on-line convex hull problem*, we are given the set of n points one point at a time. After receiving each point, we are to compute the convex hull of those points so far. We could obviously run Graham's scan once for each point, with a total running time of $O(n^2 \log n)$. Show how to solve the on-line convex-hull problem in a total of $O(n^2)$ time.

6. A group of n ghostbusters is battling n ghosts. Each ghostbuster can shoot a single energy beam at a ghost, eradicating it. A stream goes in a straght line and terminates when it hits the ghost. The ghostbusters must all fire at the same time and no two energy beams may cross to make sure that they bust all the ghosts. Assume that the positions of the ghosts and the ghostbusters are fixed points in the plane and that no three points are collinear.

   (a) Prove that for any configuration of ghosts and ghostbusters there exists such a non-crossing matching.

   (b) Show that there exist a line passing through one ghostbuster and one ghost such that the number of ghostbusters on one side of the line equals the number of ghosts on the same side. Describe how to find such a line in $O(n \log n)$ time.

   (c) Give an $O(n^2 \log n)$-time algorithm to pair ghostbusters with ghosts so that no two streams cross.