COT5520: COMPUTATIONAL GEOMETRY

Homework # 1

Due date: Sep 19, 2006, Tuesday (beginning of the class)

Your solutions should be concise, but complete, and typed (or handwritten clearly). Feel free to consult textbooks, journal and conference papers and also each other, but write the solutions yourself and cite your sources. Answer **only five** of the following six questions. Each problem is worth 20 pts.

1. Implement at least three of the convex hull algorithms discussed in class using your favorite programming language. Test your algorithms on data sets of various size (e.g., 100-10K) and of various distribution (e.g, random, convex position). Plot the computation time with respect to input size for each input distribution.

2. The region between two parallel lines is called a *slab*. The *width* of a point set P, denoted by $w(P)$ is the width of the smallest slab that contains all the points in P. Show that $w(P) = w(\mathit{ConvexHull}(P))$. For a point set P, given the $\mathit{ConvexHull}(P)$ in counterclockwise ordered representation, describe how to determine $w(P)$ in linear time.

3. A *simple polygon* is a region enclosed by a single chain of edges that does not intersect itself. Recall Graham's scan algorithm discussed in class. First, we compute a star-shaped simple polygon of the point set. Then we used a walking scheme with three markers to compute the convex hull of the star-shaped simple polygon. Describe a simple polygon for which this second stage fails to produce the convex hull. Describe an algorithm to construct the convex hull of any simple polygon in linear time.

4. Recall Chan's algorithm described in class. Given a set P of n points we first partition P into n/h subsets each of size h, where h is the size of $\mathit{ConvexHull}(P)$. Then, we compute the convex hull of each subset in $O(h \log h)$ time. Finally we use a Jarvis-type wrapping scheme which employs $O(\log h)$ time queries for each subset. The complexity of this wrapping step is $O(n \log h)$. Describe an alternative wrapping scheme that takes only $O(n)$ time.

5. Let S be a set of n disjoint triangular islands in a planar ocean. Describe a plane sweep algorithm that runs in $O(n \log n)$ time to find a set of $n - 1$ straight bridges with the following properties
   - Each bridge connects an island to another island.
   - The interiors of the bridges do not intersect each other or the islands.
   - Together they connect all islands to each other.

6. Let P be a set of n points in the plane. Two points $p, q \in P$ are said to be rectangularly visible if the rectangle with p and q as opposite corners contains no other points in P (let's assume no two points are on the same vertical line). Show that it takes $\Omega(n \log n)$ time to compute all the rectangularly visible points in time.