CIS4930-CIS6930: COMPUTATIONAL GEOMETRY

Final Exam

Date: Dec 7, 2004, Tuesday
Time: 11:45am – 1:45pm
Professor: Alper Üngör (Office CSE 430)

This is a closed book exam. No collaborations are allowed. As we agreed, you can use a cheat-sheet (one side of an A4 paper). Your solutions should be concise, but complete, and handwritten clearly. Use only the space provided in this booklet, including the even numbered pages. Feel free to give reference to the algorithms, definitions and concepts discussed in class rather than describing them in detail. You should answer all the questions to get full credit.

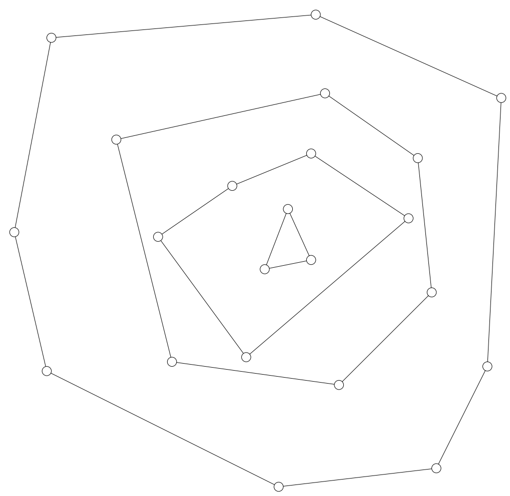
GOOD LUCK!

Your name: ____

	Credit	Max
Problem 1		10
Problem 2		10
Problem 3		15
Problem 4		15
Problem 5		15
Problem 6		15
Problem 7		20
Total		100

1. [10 points] Mrs. Obtuse is an architect and would like to design a museum building where all the walls meet at an obtuse angle, i.e., both interior and exterior angles are strictly larger than $\pi/2$. Construct an example to show that $\lfloor n/4 \rfloor$ cameras might be necessary to guard her museum design with n piecewise-linear walls.

2. [10 points] Given a set P of points in the plane, define the convex layers of P as follows: The first convex layer of P is just the convex hull of P. For all $i > 1$, the ith convex layer is the convex hull of P after the vertices of the first $i-1$ layers have been removed. Describe an $O(n^2)$-time algorithm to find all convex layers of a given set of n points.



3. [15 points] The *MinMax Area* triangulation of a point set P has the minimal largest area triangle among all possible triangulations of P.

   (a) Prove by example that the MinMax Area triangulation is not equal to the Delaunay triangulation.

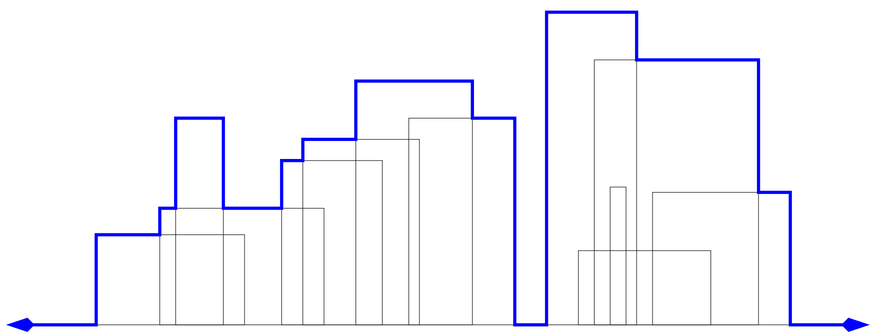
   (b) How large the following ratio can get?

$$\frac{\text{area of the largest triangle in the Delaunay triangulation}}{\text{area of the largest triangle in the MinMax Area triangulation}}$$

4. [15 points] A series of hurricanes changed the skyline in many of the Florida cities. We need to design a fast algorithm for computing the skyline of these cities. Input to your algorithm is the height, width, and left x-coordinate of n rectangles. Skyline is the chain of line segments which form the upper hull of these rectangles.

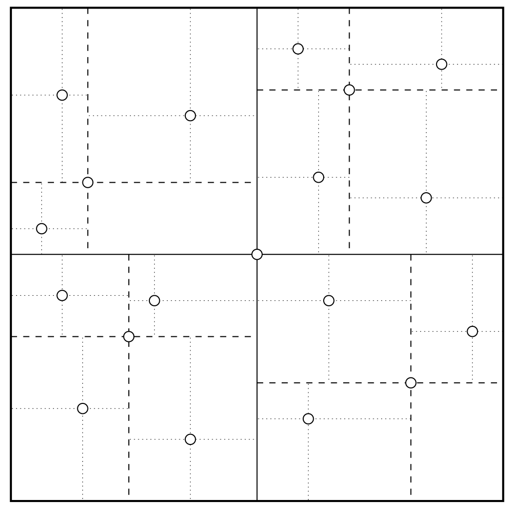
   (a) Give a tight upper bound on the complexity of the skyline?

   (b) Describe and anaylze an efficient algorithm to compute the skyline.



5. [15 points] Prove that $\Omega(n \log n)$ is a lower bound for computing the convex hull of a set of points in $\mathbb{R}^2$ in a model of computation where sorting takes $\Omega(n \log n)$ time. (Hint: Describe a transformation from a set of numbers $a_1, a_2, \ldots, a_n$, to a set of points in $\mathbb{R}^2$.)

6. [15 points] Let P be a set of points in the plane, no two sharing an x or y-coordinate. A *point quadtree* is formed by choosing a pivot point $p \in P$, splitting P into four subsets by the horizontal and vertical lines through p, and recursively constructing a point quadtree for each (non-empty) subset. A point quadtree is *perfectly balanced* if its four subtrees are perfectly balanced and its largest subtree has at most one more node than its smallest subtree.



(a) Show that a perfectly balanced point quadtree can be used to answer orthogonal counting and reporting queries in time $O(\sqrt{n})$ and $O(\sqrt{n}+k)$ time respectively.

(b) Describe a set of points for which no perfectly balanced point quadtree exists.

7. [20 points] Given a set L of n lines in the plane, describe an algorithm which computes an axis-aligned rectangle that contains all the vertices of the arrangement of L in $O(n \log n)$ time.