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Differentiated congestion pricing of urban transportation networks with vehicle-tracking technologies ☆

Mahmood Zangui [a], Yafeng Yin [a,*], Siriphong Lawphongpanich [b], Shigang Chen [c]

[a] *Department of Civil and Coastal Engineering, University of Florida, Gainesville, FL 32611-6595, USA*
[b] *Department of Industrial and Systems Engineering, University of Florida, Gainesville, FL 32611-6595, USA*
[c] *Department of Computer and Information Science and Engineering, University of Florida, Gainesville, FL 32611-6595, USA*

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A B S T R A C T

This paper explores a new type of congestion pricing that differentiates users with respect to their travel characteristics or attributes, and charges them different amounts of toll accordingly. The scheme can reduce the financial burden of travelers or lead to more substantial reduction of congestion. Given that the scheme requires tracking vehicles, an incentive program is designed to mitigate travelers' privacy concerns and entice them to voluntarily disclose their location information.

1. Introduction

Price discrimination or differentiation is an economic concept defined by Dupuit (1894) as a situation where identical products are sold for different prices (Varian, 1992). Pigou (1932) later classified price discrimination into three categories. First-degree price discrimination is the case where everyone pays his or her maximum willingness-to-pay for the product. If the unit price of the product depends on the number of units being purchased, it is classified as second degree. Lastly, third-degree discrimination means that the price of one unit of the product can be different for different type of users.

Price discrimination is not uncommon in the transportation market. A good example for second-degree discrimination is transit fare, when, e.g., a two-way ticket is cheaper than two one-way tickets, or the price of a daily pass is independent of the number of rides taken by a passenger within one day. Moreover, some transit agencies differentiate travelers by age and collect different fares for kids, students, adults and elder people, which is an example of third-degree discrimination. Previous studies have discussed price discrimination in the context of congestion pricing. Wang et al. (2011) and Lawphongpanich and Yin (2012) investigated nonlinear pricing, which is essentially an instance of second-degree discrimination where the amount of toll depends on, not strictly proportional to, the distance traveled inside a tolling area. A case of third-degree discrimination is investigated in Holguin-Veras and Cetin (2009), which differentiated users based on their vehicle type. Others, e.g., (Small and Yan, 2001; Yang and Zhang, 2002; Yang and Huang, 2004; Yin and Yang, 2004), differentiated users based on their values of travel time. De Palma and Lindsey, 2004 compared the effect of toll differentiation based on the value of time

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* Corresponding author. Tel.: +1 352 392 9537; fax: +1 352 392 3394.
E-mail address: yafeng@ce.ufl.edu (Y. Yin).

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and vehicle type on welfare. As pointed out by Pigou (1932), third-degree price discrimination generally requires an ability to distinguish different customer groups, i.e., there must be some observable attributes associated with each group, unless the pricing scheme possesses a self-selection mechanism. Given that the value of time is not directly observable, it is not surprising to find little practice of price differentiation with respect to the value of time.

This paper discusses another third-degree differentiated pricing scheme that differentiates travelers with respect to their travel characteristics or attributes, i.e., origins, destinations, or paths that they traverse between their origins and destinations. Although similar schemes may have been implemented in closed networks, e.g., tolled freeways, to our best knowledge, it has not been explored in an open, urban network environment for the purpose of congestion mitigation. We note that the advancements of vehicle-tracking and telecommunication technologies have technically enabled such a price differentiation.

The contributions of this paper are threefold. First, we use numerical examples to demonstrate the potentials of price differentiation with respect to origin, origin–destination (OD) pair or path. The examples show that in a first-best network condition where all the links are tollable, differentiated pricing can substantially reduce travelers' financial burden; in a second-best environment where only some links are tollable, it helps achieve a lower level of congestion. Second, we formulate optimization models to determine optimal differentiated pricing schemes for general networks. Third and more importantly, recognizing that price differentiation with respect to travel characteristics may compromise travelers' location privacy, we propose an approach of modeling privacy, and then design an incentive program to provide incentives for travelers to reveal their travel information and voluntarily participate in differentiated pricing. Such an opt-in program is designed to create a win–win situation for both travelers and society.

The remainder of this paper is organized as follows. Section 2 discusses different types of differentiated pricing and their formulations, and presents numerical examples to make a case for differentiated pricing. Section 3 discusses the location privacy issue associated with differentiated schemes, and proposes an approach of modeling privacy. Section 4 is dedicated to the development of an incentive program for differentiated pricing. Lastly, Section 5 concludes the paper and discusses another way to mitigate travelers' privacy concerns.

2. Differentiated pricing schemes

Differentiated pricing schemes we discuss in this paper include origin-specific, OD-specific and path-based. As their names suggest, travelers on the same link will be charged differently, with respect to their respective origin, OD pair or path. Intuitively, these schemes are more flexible than traditional anonymous tolling. Mathematically, they can be viewed as different levels of relaxation to anonymous schemes.

To facilitate the presentation, we label the differentiation level of anonymous pricing as zero, and subsequently the levels of differentiation for origin-specific, OD-specific and path-based pricing as one, two and three, respectively.

2.1. Notation

Let $G(N,A)$ denote a transportation network, where N is the set of nodes and A is the set of directed links. Index a is used to denote a link, which is also represented by its end nodes $i, j \in N$, i.e., $(i,j) = a$. For link a, x_a and γ_a are its aggregate flow and toll, respectively. The latter is expressed in the unit of time for the sake of simplicity. Let $W \subseteq N \times N$ be the set of OD pairs with strictly positive demand, w be the index of its elements and d_w be the demand of OD pair w. For every OD pair $w \in W$, $o(w)$ represents its origin. The set of all paths connecting OD pair w is denoted by P_w with its elements being indexed by p. A binary parameter δ represents the link-path incidence, i.e., if link a is on path p, then δ_{ap} is one; otherwise zero. For every path p, f_p and π_p denote its flow and toll, respectively. Again, the toll is represented in the unit of time. Also, $t_p(.)$ and $t_a(.)$ are the travel time for path p and link a, respectively. For second-best pricing, the set of tollable links is denoted by Ψ, and its complement set $\overline{\Psi}$ includes all the untollable links.

2.2. Formulations

As aforementioned, path-based scheme has the highest level of differentiation, because the origin or destination of a trip can be easily determined from the path utilized by the trip. Hence, a general path-based formulation is used in this paper to describe all three differentiation schemes. Notice that origin-specific and OD-specific pricing are link-based schemes, and thus the toll of each path is the sum of tolls on links comprising the path. In contrast, path-based tolls are determined for specific paths.

We first discuss a first-best network condition where all links are tollable. In such an environment, even with the lowest level of price differentiation, i.e., anonymous tolling, congestion pricing is able to induce system optimum and replicate system optimum link flows (e.g., Hearn and Ramana, 1998; Lou et al., 2010). Consequently, the benefit of price differentiation can only be reflected on a secondary objective. In this paper, we choose revenue minimization as the secondary objective because it represents a financial burden to the traveling public. Below, we formulate a program for finding a first-best path-based pricing scheme to minimize the total toll revenue:

$$\min \quad \sum_{w \in W} \sum_{p \in P_w} \pi_p f_p \tag{1}$$

$s.t.$

$$\sum_{p \in P_w} f_p = d_w \quad \forall w \in W \tag{2}$$

$$f_p(t_p(f) + \pi_p - \lambda_w) = 0 \quad \forall p \in P_w, w \in W \tag{3}$$

$$t_p(f) + \pi_p - \lambda_w \geqslant 0 \quad \forall p \in P_w, w \in W \tag{4}$$

$$f_p \geqslant 0 \quad \forall p \in P_w, w \in W \tag{5}$$

$$\pi_p \geqslant 0 \quad \forall p \in P_w, w \in W \tag{6}$$

$$\sum_{w \in W} \sum_{p \in P_w} \delta_{ap} f_p = \bar{x}_a \quad \forall a \in A \tag{7}$$

where $\bar{x}_a$ is the system optimum link flow on link a. In the above, the objective function is to minimize total toll revenue. Constraint (2) is to ensure flow conservation; Constraints (3) and (4) are tolled user equilibrium conditions; Constraints (5) and (6) specify non-negative path flow and toll, and the last constraint requires link flows to replicate system optimum link flows.

The above formulation can be easily modified for the other two differentiated schemes. In origin-specific and OD-specific schemes, tolls are imposed on links, but can be different for different origins or OD pairs. In our formulation, we associate a superscript to toll variables, γ, to differentiate tolls. Subsequently, adding the following constraints to the above model yields a formulation for origin-specific pricing:

$$\pi_p = \sum_{a \in A} \delta_{ap} \gamma_a^{o(w)} \quad \forall p \in P_w, w \in W \tag{8}$$

$$\gamma_a^{o(w)} \geqslant 0 \quad \forall a \in A, w \in W \tag{9}$$

where $\gamma_a^{o(w)}$ is the toll on link a for users from origin $o(w)$.

Similarly, the formulation for OD-specific pricing can be obtained by adding the following constraints:

$$\pi_p = \sum_{a \in A} \delta_{ap} \gamma_a^{w} \quad \forall p \in P_w, w \in W$$

$$\gamma_a^{w} \geqslant 0 \quad \forall a \in A, w \in W$$

where γ_a^w is the toll on link a for users of OD-pair w. It is worth mentioning that Constraint (6) becomes redundant and can be removed from the formulations for both origin- and OD-specific schemes.

We now consider a second-best network condition where not all the links are tollable. In this case, anonymous tolling may not induce system optimum and thus price differentiation provides additional flexibility to further reduce system travel time. Below we present a formulation to obtain a second-best origin-specific pricing scheme that minimizes system travel time:

$$\min \quad \sum_{w \in W} \sum_{p \in P_w} t_p(f) f_p \tag{10}$$

$s.t.$

$$(2), (3), (4), (5), (8), (9)$$

$$\gamma_a^{o(w)} = 0 \quad \forall w \in W, a \in \overline{\Psi} \tag{11}$$

The OD-specific formulation can be developed similarly. Notice that because path-based pricing does not impose tolls on links, it becomes irrelevant in the second-best network condition, which is considered in this paper to be the situation where not all links are tollable.

Comparing the above with the first-best formulations, Eq. (7) is no longer included because system optimum link flows may not be achievable. In addition, because only specific links can be tolled, Constraint (11) is added. We further note that link-based formulations for the origin-specific and OD-specific schemes exist, but we do not present them to keep the paper concise.

Because of Constraints (3)–(5), the formulations presented above all belong to the class of mathematical programs with complementarity constraints (MPCC). These problems are non-convex and standard stationary conditions, i.e., KKT conditions, may not hold for them because they do not satisfy Mangasarian-Fromovitz constraint qualification (Scheel and Scholtes, 2000). Many solution algorithms have been proposed for MPCC (see, e.g. Luo et al., 1996 and references cited therein). However, some only work well for small and medium problems while others, especially those based on solving equivalent nonlinear programs (e.g., Lawphongpanich and Yin, 2010), can handle larger problems. More effective algorithms may be developed to solve the above formulations by exploring special properties or structures that they may possess. For example, Zangui et al. (2013) reformulated the first-best path-based pricing problem as a linear integer program that can be solved to global optimality. In this paper, we approximate complementarity constraints to be nonlinear inequality

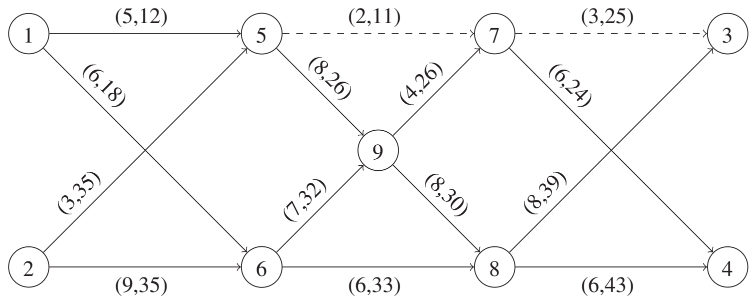

Fig. 1. Nine-node network.

Table 1
Differentiated pricing for nine-node network with all links tollable (unit: min).

Tolling scheme	Toll revenue		OD generalized travel cost			
	Amount	Reduction (%)	[1,3]	[1,4]	[2,3]	[2,4]
User equilibrium	–	–	24.9	23.8	24.3	25.1
Anonymous	887.6	0	30.6	29.2	33.0	31.6
Origin-specific	311.6	65	23.4	29.3	25.8	24.4
OD-specific	295.6	67	23.4	22.0	25.8	27.6
Path-based	263.6	70	23.4	22.0	29.0	24.4

Table 2
Second-best differentiated pricing for nine-node network (unit: min).

Tolling scheme	Total travel time		OD generalized travel cost			
	Amount	Saving (%)	[1,3]	[1,4]	[2,3]	[2,4]
User equilibrium	2455.9	0	24.9	23.8	24.3	25.1
Anonymous	2361.2	46.9	25.8	24.9	25.1	25.9
Origin-specific	2306.1	74.2	24.3	24.2	27.1	25.7
OD-specific	2281.7	86.2	24.4	22.9	26.8	25.3
System optimum	2253.9	100	–	–	–	–

constraints and solve a MPCC as a nonlinear program (Scheel and Scholtes, 2000). Since the nonlinear program is non-convex, we solve it with multiple initial solutions and present the best-obtained solution.

2.3. Illustrative examples

We now demonstrate the potentials of differentiated pricing schemes on a nine-node network. Fig. 1 shows the network with four OD pairs [1,3], [1,4], [2,3], and [2,4], whose demands are 10, 20, 30 and 40, respectively. The link performance functions are of the following form:

$$t_a(x_a) = T_a\left(1 + 0.15\left(\frac{x_a}{b_a}\right)^4\right)$$

where T_a and b_a are provided in Fig. 1 as (T_a, b_a) near each link.

Table 1 presents the results[1] of different levels of differentiation when all links are tollable. The second and third columns show the minimum toll revenue of each scheme, and the percent reduction as compared to the anonymous scheme. It can be observed that the toll revenues for all differentiated schemes are substantially lower than that of anonymous pricing. Moreover, as the level of differentiation increases, the revenue decreases. Particularly, price differentiation with respect to path yields a 70% reduction in revenue. The last four columns present the equilibrium travel cost for each OD pair. Observe that other than OD-pair [1,4] under origin-specific scheme, the travel costs under differentiated schemes are less than those under the anonymous scheme, suggesting that differentiated pricing may be more appealing to individual travelers in this network.

Table 2 presents the results of solving second-best differentiated pricing for the nine-node network, when only links (5,7) and (7,3) are tollable. In this table, the second column shows the total system travel time under each tolling scheme.

[1] Results are the best-obtained ones, but likely local optima. This note applies to other tables in this paper.

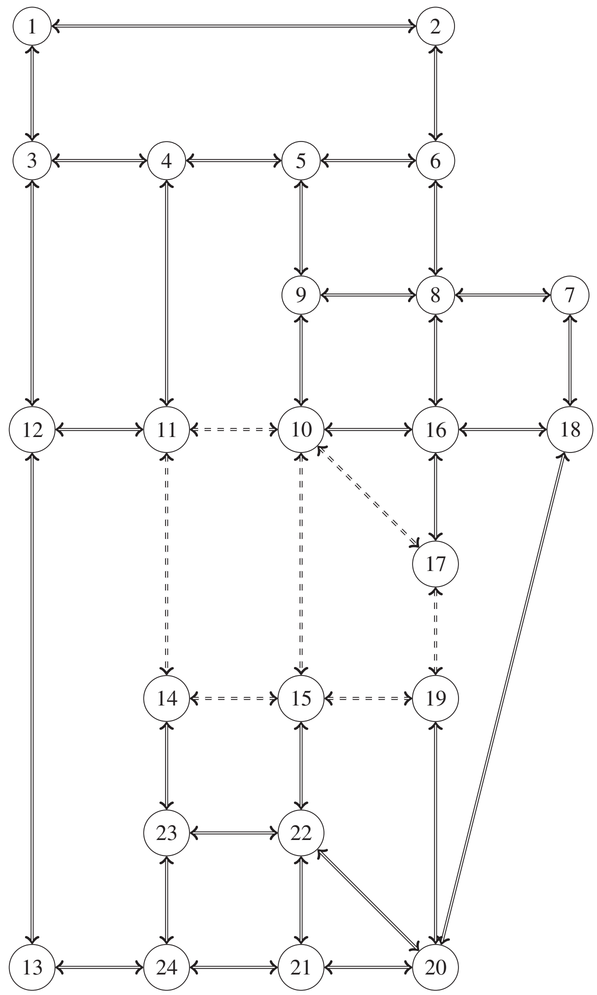

Fig. 2. Sioux Falls network.

Table 3
Differentiated pricing for Sioux Falls network (unit: 10^3 h).

Tolling scheme	First-best		Second-best	
	Min. revenue	Reduction (%)	Travel time	Saving (%)
Anonymous	20.666	0.00	74.043	26.55
Origin-specific	0.750	96.37	73.060	60.93
OD-specific	0.616	97.02	72.997	63.13
Path-based	0.182	99.12	–	–
User equilibrium	–	–	74.802	0.00
System optimum	–	–	71.943	100.00

Knowing that system optimum yields the smallest system travel time, we present the third column as the ratio between travel time reduction from user equilibrium and the maximum possible reduction, i.e., the difference in travel times of user equilibrium and system optimum. It is evident that price differentiation leads to additional travel time reduction. Specifically, even with only two links being tollable, the OD-specific tolling scheme achieves 86.2% of the maximum possible reduction, a reduction achieved by a first-best pricing scheme that may toll all links.

We also solved for differentiated schemes on the Sioux Falls network from LeBlanc et al. (1975) as shown in Fig. 2 and the results are presented in Table 3. For second-best pricing, only the dashed links in Fig. 2 are assumed to be tollable. Table 3 shows that first-best differentiated pricing yields a substantial reduction in toll revenue, while minimizing system travel time. Similarly, the second-best pricing schemes offer promising results. Using system optimum as the benchmark, the additional travel time under user equilibrium is 2.859. The origin-specific scheme can reduce the additional time to 1.117, which is equivalent to a 60.93% reduction. Compared to anonymous tolling, origin-specific tolls can achieve approximately twice travel time reduction.

In general, for both the first-best and second-best conditions, higher levels of differentiation lead to more favorable results. On the other hand, differentiated pricing schemes are more difficult to implement. One needs to consider such a trade-off to determine whether a higher level of differentiation is worth implementing or not on a particular network.

3. Location privacy

One of the major reasons for the implementation difficulty of differentiated pricing is potential violation of motorists' location privacy. Location privacy is defined as the ability to prevent other parties from learning one's current or past location (Beresford and Stajano, 2003). The issue of location privacy commonly arises when offering a service requires some sort of location data. The issue has been mostly studied for situations where mobile applications or computer programs need to know the location of a user (e.g., Cvrcek et al., 2006; Ban and Gruteser, 2012). In the transportation field, recent studies, e.g., (Sun et al., 2013), have investigated approaches to simultaneously guarantee privacy protection and data needs for traffic modeling applications.

The traditional way of manually collecting toll preserves location privacy almost completely. Needless to say, it is not an efficient way to collect toll, as vehicles have to stop and pay. Electronic toll collection (ETC) systems have been built to make toll collection more efficient, but the way they currently operate may compromise motorists' privacy rights (Sager, 1998). The systems often link motorists' accounts and record locations and times of transactions (e.g., the Sunpass prepaid toll program in Florida (Florida Department of Transportation, 2012)). If toll gantries are ubiquitous, the recorded transaction information may impinge on the privacy rights of motorists. However, those who are concerned about their location privacy have the option to pay the toll by cash and avoid risk of privacy disclosure. Moreover, for anonymous link-based tolling, it is possible to design a privacy-preserving ETC system (e.g., Cavoukian, 1998; Balasch et al., 2010).

Unfortunately, it is difficult, if not impossible, to design a privacy-preserving differentiated pricing system, because the system requires the knowledge of travelers' location information, such as the origin and destination of each trip for an OD-specific scheme. Golle and Partridge (2009) and Krumm (2009) pointed out that the home/work location data, even if they are anonymous, can be used to identify individuals. In addition to this, the sole fact of being tracked by the tolling system can cause inconvenience or discomfort. All these privacy concerns need to be addressed.

On the other hand, there have been some indications that motorists, some at a price, are willing to provide location information with the understanding that it will not be published and/or misused. For example, in the Travel Choices Study completed by the Puget Sound Regional Council (2008), each participant was given a $1016 debit account with a GPS-based onboard unit installed on his or her car. This unit tracks and records when and where the participants drive and deducts tolls from the account. The money remaining in each account at the end of the study was given to the study participant. In this example, location information was collected for the purpose of tolling and with full knowledge of study participants. We surmise that the participants may be attracted to the $1016 incentive when joining the study.

Empirical experiments in the literature have proved that individuals value their location privacy differently. They can be grouped into categories of privacy unconcerneds, privacy pragmatists, and privacy fundamentalists (Krumm, 2009). The first group do not care about location privacy and are insensitive to the negative consequences of location leak. The second group are willing to reveal their location for a, sometimes very small, price, while the last group highly value and strive to protect their location privacy. Mathematically, we can use a distribution to represent different individual valuations of privacy across the population. Acquisti et al. (2000) suggested a U-shaped distribution, but cautioned that the value of privacy can be very malleable and many non-normative factors may affect its distribution (also see Cvrcek et al., 2006). Hence, we do not base our models on any specific, but a general distribution for the value of privacy. Nevertheless, it is important to understand the implication of a proposed distribution. For example, although logistic distributions offer computational advantages as we have seen in the choice modeling, such distributions imply that some users will have a negative value of privacy, an unjustifiable assumption.

Fig. 3 illustrates more reasonable uniform and exponential distributions, both with a mean of two. In this figure, $U(0,4)$ denotes a uniform distribution between 0 and 4, and $EXP(0.5)$ is an exponential distribution with a parameter of 0.5. Notice that the exponential distribution is more clustered around smaller values, which implies that more users value their privacy less. However, exponential distributions with higher mean values become more evenly distributed. Also notice that the span

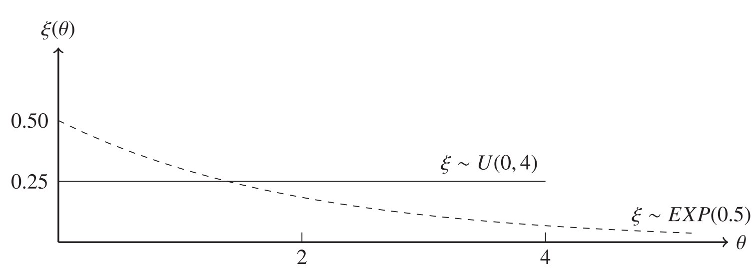

Fig. 3. Uniform and exponential distributions with same mean, $E(\theta) = 2$.

for an exponential distribution is all non-negative real numbers, while the uniform distribution is bounded on both sides. So, uniform distribution implies that the value of privacy of travelers is evenly distributed and has an upper bound. On the other hand, the exponential distribution suggests that some travelers are extreme privacy fundamentalists and will not disclose their locations at any price.

3.1. Modeling privacy

We now use origin-specific pricing as an example for modeling privacy. Denote the travel cost between OD pair w under the scheme as $\lambda_{w,1}$, which consists of travel time and toll. Since they are being tracked, motorists incur additional cost for the loss of their location privacy, which we call privacy cost. Mathematically, the full cost for a traveler between OD pair w under origin-specific pricing is $\lambda_{w,1} + \beta$, where β is a random variable representing the value of privacy, which is also expressed in the unit of time for simplicity. In this paper, we assume the distribution of value of privacy is the same for travelers from each OD pair, but this assumption can be easily relaxed.

Suppose the value of privacy follows a known distribution ξ, i.e., $\beta \sim \xi$. Define $\Xi(\theta) = \int_0^\theta \xi(z)dz$ as the cumulative distribution function associated with the value of privacy, i.e., Prob $(\beta \leqslant \theta) = \Xi(\theta)$. Denote the travel cost between OD pair w under anonymous tolling as $\lambda_{w,0}$ and let $\theta_w = \lambda_{w,0} - \lambda_{w,1}$. If θ_w is negative, no individual from this OD pair prefers the origin-specific scheme. As we observe from Tables 1 and 2, θ_w is likely positive. In this case, travelers who value their privacy less than θ_w would prefer the origin-specific scheme to the anonymous one, while those with higher value of privacy would prefer anonymous tolling. The percentage of the former is $\Xi(\theta_w)$ while it is $1 - \Xi(\theta_w)$ for the latter. Fig. 4 illustrates this situation for a hypothetical distribution of the value of privacy, where the shaded area represents the percentage of travelers who will be better off and thus prefer the origin-specific scheme. Their total privacy cost, denoted as $PC_w(\theta_w)$, can be computed as follows: $PC_w(\theta_w) = \int_0^{\theta_w} d_w \xi(z) z dz$, where d_w is the total demand between OD pair w. Define $E(\beta;\theta_w) = \int_0^{\theta_w} z\xi(z)dz$, and the equation is thus written as $PC_w(\theta_w) = d_w E(\beta;\theta_w)$.

The calculations of $\Xi(\theta)$ and $E(\beta;\theta)$ involve integration. If the value of privacy follows a uniform or an exponential distribution, the integrals will have a closed form. In a general case where the integrals do not have a closed form, we need to compute them via numerical integration methods, such as Riemann sum as follows:

$$\Xi(\theta) = \int_0^\theta \xi(z)dz = \frac{1}{n}\theta\sum_{i=1}^{n}\xi\left(\frac{i\theta}{n}\right)$$

$$E(\beta;\theta) = \int_0^\theta z\xi(z)dz = \frac{1}{n}\theta\sum_{i=1}^{n}\left(\frac{i\theta}{n}\xi\left(\frac{i\theta}{n}\right)\right)$$

where n is the number of bars used to approximate the area. Choosing a larger n would result in a higher precision.

3.2. Privacy analysis of differentiated schemes

In this section, we examine the results of the nine-node network in Section 2.3 from a privacy perspective. Table 4 calculates the percentages of travelers between each OD pair who would benefit from origin-specific pricing after considering privacy cost, i.e., $100\Xi(\theta_w)$, under different hypothetical distributions for the value of privacy.

The third row of Table 4 shows the saving of time and toll for travelers between each OD pair, i.e. $\theta_w = \lambda_{w,0} - \lambda_{w,1}$. It can be observed that, even if the average value of privacy is high, some travelers still benefit from differentiated schemes. However, the percentage decreases as the average value of privacy increases. Also observe that when travel cost saving is small, exponential distributions predict higher percentages of users who will benefit from differentiated schemes, because the distributions are more clustered around smaller values.

Apparently, the savings of time and toll that some travelers enjoy from differentiated schemes are offset by the loss of their privacy. Section 4 presents a way to take advantage of the potentials of differentiated pricing, while allowing those concerned travelers to maintain their privacy.

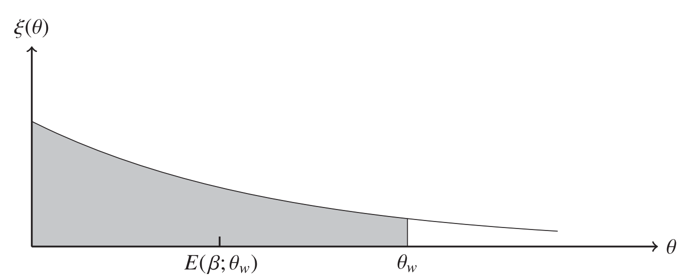

Fig. 4. Expected privacy cost.

Table 4
Percentage of travelers who benefit from origin-specific pricing on nine-node network.

Network condition	First-best				Second-best			
OD pair	[1,3]	[1,4]	[2,3]	[2,4]	[1,3]	[1,4]	[2,3]	[2,4]
Travel cost saving	7.2	−0.1	7.2	7.2	1.5	0.7	−2.0	0.2
$\beta \sim U(0,4)$	100.00	0.00	100.00	100.00	37.50	17.50	0.00	5.00
$\beta \sim U(0,8)$	90.00	0.00	90.00	90.00	18.75	8.75	0.00	2.50
$\beta \sim U(0,16)$	45.00	0.00	45.00	45.00	9.37	4.38	0.00	1.25
$\beta \sim EXP(0.500)$	97.27	0.00	97.27	97.27	52.76	29.53	0.00	9.52
$\beta \sim EXP(0.250)$	83.47	0.00	83.47	83.47	31.27	16.05	0.00	4.88
$\beta \sim EXP(0.125)$	59.34	0.00	59.34	59.34	17.10	8.38	0.00	2.47

4. Addressing privacy concerns with an incentive program

Recognizing that some may benefit from differentiated schemes while others with higher value of privacy may be better off under anonymous tolling, we propose to develop an incentive program for travelers to opt into differentiated pricing. More specifically, a hybrid of anonymous and differentiated pricing schemes will be implemented on the network. Travelers who choose to reveal their location information will pay differentiated tolls while those who do not disclose their information pay anonymous tolls.

Since travel costs (time plus toll) in differentiated schemes are generally less than those in the anonymous scheme, the cost savings can be viewed as incentives for drivers to participate in differentiated pricing. Although other incentives, such as subsidies or credits, can be provided, below we focus on designing anonymous and differentiated tolls in the hybrid scheme and allowing for the cost savings as incentives. The overall goal of this hybrid scheme is to create a win–win situation for both users and society.

4.1. Design of incentive program

As an example, we design the incentive program for a hybrid of origin-specific and anonymous tolls. The formulations for other hybrid schemes can be developed with some straightforward modifications. Hence, we do not present them to keep the paper concise.

It is reasonable to assume all the motorists who are better off under an origin-specific scheme will opt into this scheme. Thus, the number of these motorists will be $d_{w,1} = \Xi(\lambda_{w,0} - \lambda_{w,1})d_w$. Travelers who choose the anonymous scheme will not incur any privacy cost. Hence, the total privacy cost for travelers between OD pair w is equal to $PC_w(\lambda_{w,0} - \lambda_{w,1})$, as defined in Section 3.1.

The following constraints define the feasible region of the problem:

$$d_{w,0} + d_{w,1} = d_w \quad \forall w \in W \tag{12}$$
$$d_{w,1} = \Xi(\lambda_{w,0} - \lambda_{w,1})d_w \quad \forall w \in W \tag{13}$$
$$\sum_{p \in P_w} f_{p,c} = d_{w,c} \quad \forall w \in W, c \in C \tag{14}$$
$$f_{p,c}(t_p(f) + \pi_{p,c} - \lambda_{w,c}) = 0 \quad \forall p \in P_w, w \in W, c \in C \tag{15}$$
$$t_p(f) + \pi_{p,c} - \lambda_{w,c} \geqslant 0 \quad \forall p \in P_w, w \in W, c \in C \tag{16}$$
$$\lambda_{w,0} \geqslant \lambda_{w,1} \quad \forall w \in W \tag{17}$$
$$f_{p,c} \geqslant 0 \quad \forall p \in P_w, w \in W, c \in C \tag{18}$$
$$\pi_{p,c} \geqslant 0 \quad \forall p \in P_w, w \in W, c \in C \tag{19}$$
$$\pi_{p,0} = \sum_{a \in A} \delta_{ap}\gamma_a \quad \forall p \in P_w, w \in W \tag{20}$$
$$\gamma_a \geqslant 0 \quad \forall a \in A \tag{21}$$
$$\pi_{p,1} = \sum_{a \in A} \delta_{ap}\gamma_a^{o(w)} \quad \forall p \in P_w, w \in W \tag{22}$$
$$\gamma_a^{o(w)} \geqslant 0 \quad \forall a \in A, w \in W \tag{23}$$

where $C = \{0,1\}$. Constraints (12) and (13) split the demand for each OD pair. Constraint (14) ensures flow balance. The tolled user equilibrium is guaranteed by Constraints (15) and (16). Constraint (17) requires travel cost (time plus toll) in the origin-specific scheme to be less than that in the anonymous scheme. Constraints (20) and (22) make the toll on each path to be equal to the sum of link tolls. Denote the feasible region defined by the above constraints as Φ.

We first discuss the problem of finding the optimal hybrid scheme in the first-best network setting where all the links are tollable. In this situation, we are interested in replicating the flow distribution with minimum system travel time, i.e., $\bar{x}$, as well as minimizing the user cost as a secondary objective. The following is the total (full) user cost:

$$\sum_{w\in W}(PC_w(\lambda_{w,0}-\lambda_{w,1})+\sum_{p\in P_w}(\pi_{p,0}f_{p,0}+\pi_{p,1}f_{p,1}))+\sum_{a\in A}x_a t_a(x_a)$$

Since $x=\bar{x}$ ought to be achieved in first-best pricing, the last term is a constant and can be omitted from the optimization. Consequently, we have the following formulation for finding an optimal hybrid scheme in a network with all links being tollable:

$$\min \quad \sum_{w\in W}(PC_w(\lambda_{w,0}-\lambda_{w,1})+\sum_{p\in P_w}(\pi_{p,0}f_{p,0}+\pi_{p,1}f_{p,1})) \tag{24}$$
$$s.t.$$
$$(f,d,\pi,\lambda)\in\Phi$$
$$\sum_{w\in W}\sum_{p\in P_w}\delta_{a,p}(f_{p,0}+f_{p,1})=\bar{x}_a \qquad \forall a\in A$$

where the last constraint is to ensure the link flows to be the least-system-time flows.

We now consider a second-best situation when only some links are tollable. In this situation, we attempt to minimize total system cost, which differs from the above total (full) user cost by the toll revenue, because the revenue is not a cost for the system but a transfer from travelers to the government. The problem of finding an optimal hybrid scheme can be formulated as follows:

$$\min \quad \sum_{w\in W}(PC_w(\lambda_{w,0}-\lambda_{w,1})+\sum_{p\in P_w}t_p(f)f_p) \tag{25}$$
$$s.t.$$
$$(f,d,\pi,\lambda)\in\Phi$$
$$\gamma_a^{o(w)}=0 \qquad \forall w\in W, a\in\overline{\Psi}$$

The last constraint ensures that only tollable links can have positive amount of toll.

Notice that the above formulations are path-based, and solving them requires path enumeration. However, it is possible to formulate them as link-based models. We use the above path-based formulations to facilitate the presentation.

4.2. Numerical examples

The proposed models for designing the incentive program of the origin-specific scheme were implemented on the nine-node and Sioux Falls networks. Each model was solved for both uniform and exponential distributions of value of privacy, each with three different expected values.

Table 5 presents the results on the nine-node network of Fig. 1 with all the links being tollable. The performances of the hybrid schemes are also compared with those of the anonymous and origin-specific tolls when implemented separately.

As pointed out previously, origin-specific pricing can reduce the toll revenue significantly in a first-best network condition. However, this reduction comes with a price of violating travelers' privacy. Since origin-specific schemes require all the users to reveal their origin information, the privacy cost is equal to the expected value of privacy multiplying by the total demand. The privacy cost increases as travelers value their privacy more, eventually causing the total user cost under origin-specific pricing to be larger than that under anonymous tolling, when the expected value of privacy is equal to 8. In contrast, the hybrid scheme offers an option for travelers of high value of privacy to remain anonymous. Such a self-selection mechanism leads to much smaller loss of privacy and subsequently less total user cost. Interestingly, in this example, the hybrid schemes also lead to less amount of toll revenue than their origin-specific counterparts. However, this observation need not be generally true.

Table 5
Comparison of different schemes on nine-node network (all links tollable. Unit: min).

Pricing scheme	Distribution of β	$E(\beta)$	Toll revenue	Privacy cost	Total user cost
Anonymous	–	–	887.60	0.00	887.60
Origin-specific	–	2	311.60	200.00	511.60
	–	4	311.60	400.00	711.60
	–	8	311.60	800.00	1111.60
Hybrid	$U(0,4)$	2	247.82	28.46	276.28
	$U(0,8)$	4	235.25	58.47	293.72
	$U(0,16)$	8	218.10	113.92	332.02
	$EXP(0.500)$	2	249.84	17.49	267.33
	$EXP(0.250)$	4	236.16	34.57	270.73
	$EXP(0.125)$	8	210.59	69.16	279.75

Table 6
Comparison of different schemes on nine-node network (two tollable links. Unit: min).

Pricing scheme	Distribution of β	$E(\beta)$	Travel time	Privacy cost	Total system cost
Anonymous	–	–	2361.16	0.00	2361.16
Origin-specific	–	2	2306.10	200.00	2506.10
	–	4	2306.10	400.00	2706.10
	–	8	2306.10	800.00	3106.10
Hybrid	$U(0,4)$	2	2291.79	9.13	2300.92
	$U(0,8)$	4	2296.76	13.08	2309.84
	$U(0,16)$	8	2304.63	17.57	2322.20
	$EXP(0.500)$	2	2291.45	5.82	2297.27
	$EXP(0.250)$	4	2293.47	9.56	2303.04
	$EXP(0.125)$	8	2299.10	13.30	2312.40

Table 7
Second-best hybrid schemes on Sioux Falls network.

Pricing scheme	Distribution of β	$E(\beta)$ (h)	Travel time (10^3 h)	Privacy cost (10^3 h)	Total system cost (10^3 h)
Anonymous	–	–	74.043	0.000	74.043
Origin-specific	–	0.02	73.060	7.212	80.272
	–	0.04	73.060	14.424	87.474
	–	0.08	73.060	28.848	101.908
Hybrid	$U(0,0.04)$	0.02	73.294	0.118	73.412
	$U(0,0.08)$	0.04	73.421	0.138	73.421
	$U(0,0.16)$	0.08	73.591	0.163	73.753
	$EXP(50.0)$	0.02	73.272	0.086	73.357
	$EXP(25.0)$	0.04	73.355	0.106	73.461
	$EXP(12.5)$	0.08	73.455	0.163	73.618

We solved for the anonymous, origin-specific and hybrid schemes on the nine-node network when only two specific links, $(5,7)$ and $(7,3)$, are tollable. Table 6 displays the results for each scheme. As expected, in every case, the total cost under the hybrid scheme is less than those in the anonymous and origin-specific schemes. Interestingly, the hybrid schemes also yield even less total travel time than the origin-specific scheme, even though the latter is to minimize total travel time. As the feasible regions of these two models are not the same, one should not expect that the origin-specific problem always yields less total travel time than the hybrid counterpart. Because the hybrid scheme problem is to minimize the total system cost that includes total travel time as one component, and any feasible solution to the origin-specific problem can be obtained from the hybrid scheme problem by setting anonymous tolls to be sufficiently high, it can happen that the hybrid scheme problem yields less total travel time.

To demonstrate the models on a more realistic network, we solved them on the Sioux Falls network where the tollable links are the dashed ones in Fig. 2. The obtained results are presented in Table 7. Similar to the nine-node network, the privacy cost and total cost increase as the expected value of privacy increases. Also, the privacy cost and total cost under the exponential distributions is less than those associated with the uniform distributions.

The results in this section illustrate the potentials of the incentive program for origin-specific pricing. For two extreme cases, with the value of privacy being zero or infinity, the hybrid scheme yields the same results as differentiated or anonymous scheme, respectively. But, in the real world, this value should be finite and positive. Our results indicate that the performance of the incentive program is much better when the expected value of privacy is relatively low, i.e., more users are willing to reveal their information for a small amount of money (previous empirical studies seem suggest so). The incentive program also demonstrates promising results when the expected value of privacy is relatively higher. While this section only focuses on a hybrid of origin-specific and anonymous tolls, we expect other hybrids to perform favorably in a similar fashion.

5. Conclusion and discussion

This paper has explored a new class of tolling schemes that charge different amounts of toll for users with different origins, destinations, or paths. These schemes provide more flexibility than traditional anonymous pricing and the numerical examples in this paper have demonstrated that they can reduce the financial burden on motorists in a first-best network condition or lead to more travel time saving in a second-best condition.

Recognizing that the differentiated pricing may compromise travelers' privacy, we have proposed an incentive program to allow travelers to opt into the differentiated pricing, if they find the amount of incentive to be worth disclosing their location information. This self-selection mechanism allows the tolling agency to take advantages of the potentials of differentiated pricing without doing harm to travelers' privacy rights.

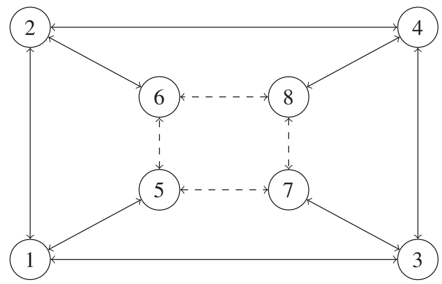

Fig. 5. An illustrative network.

Other approaches can be explored to mitigate privacy concerns associated with differentiated pricing. For instance, instead of charging users based on their true origins, the tolling agency can designate a tolling area and then charge users based on where they enter the area. Because the true origins are not revealed, this scheme may partially mitigate travelers' privacy concern. Note that this scheme is different from the traditional cordon pricing in which motorists pay a uniform toll to cross the cordon. In the refined scheme, motorists on a link within the tolling area will pay different amounts of toll depending on where they enter the tolling area. We call this scheme as a sub-network origin-specific pricing scheme. To illustrate the concept, consider the network in Fig. 5 where the tolling area consists of the dashed links. Consider three different paths, p_1:$2 \to 6 \to 5 \to 7 \to 3$, p_2:$2 \to 1 \to 5 \to 7 \to 3$, and p_3:$1 \to 5 \to 7 \to 3$. While in the original origin-specific scheme, travelers on p_1 and p_2 will pay the same amount of toll on link (5,7), they may pay different amounts of toll for traversing the link under the refined scheme, because they enter the tolled sub-network from different nodes (Nodes 6 and 5 respectively). Also, unlike in the original origin-specific pricing, motorists on p_2 and p_3 will pay the same amount of toll for link (5,7) because they both enter the sub-network from Node 5.

We implemented the sub-network origin-specific scheme for the Sioux Falls network (Fig. 2), where the tolling area consists of dashed links and nodes 10, 11, 14, 15, 17 and 19. The best design yields a system travel time of 73.215, which is slightly greater than the system travel time of 73.060 under the true origin-specific scheme. In this case, the refined sub-network origin-specific pricing is very promising. An interesting future study can be conducted to explore how to select the tolling area to achieve a similar performance as the true origin-specific tolling.

This paper assumes that OD travel demands are fixed. In transportation planning, fixed-demand models are often used to predict traffic flows during morning and evening peak periods because demands during these periods consist mainly of people commuting to and from work and are typically regarded as inelastic. However, recent evidence for Singapore, London, and Stockholm shows that travel demand during peak periods tends to be more elastic than models have predicted. It is thus worthwhile to investigate differentiated pricing under elastic demands. Future research can also be performed to explore differentiated pricing in dynamic (e.g., Lou et al., 2011; Nie and Yin, 2013) or multimodal (e.g., Wu et al., 2011; Wu et al., 2012) settings.

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