

# Truthful Auction Mechanisms with Performance Guarantee in Secondary Spectrum Markets

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**Abstract**—We study a spectrum auction problem where each request from new spectrum users has spatial, temporal, and spectral features. Our goal is to design truthful auction mechanisms that maximize either the overall social efficiency of new users (*a.k.a* buyers) or the revenue of the spectrum owner (*a.k.a* seller). Given that the optimal conflict-free spectrum allocation problem is NP-hard, this paper proposes a series of near-optimal auction mechanisms based on the following approximation techniques: linear programming (LP) relaxation, randomized rounding, derandomized rounding, monotone derandomization, and Lavi-Swamy method. Comparing with the prior art, we make two significant advances: First, our auction mechanisms are not only truthful but also provide theoretically-provable performance guarantee, an important feature that existing work under the same auction model does not have. Second, our auction mechanisms support both spatial and temporal spectral reuse, which makes the problem more challenging than existing work that deals with only spatial or temporal reuse. We perform extensive simulations to study the performance of the proposed mechanisms, and the simulation results corroborate our theoretical analysis.

**Index Terms**—Spectrum auction, performance guarantee, social efficiency maximization, revenue maximization

## 1 INTRODUCTION

THE growing demand for limited spectrum resource poses a great challenge in spectrum allocation and usage [3]. One of the most promising methods is spectrum auction, which gives incentive for a **spectrum owner** (*a.k.a* seller) to sublease spectrum to **new users** (*a.k.a* buyers). The design of spectrum auction mechanisms faces two major challenges. First, spectrum channels can be reused in spatial, temporal, and spectral domains. We show that allocating buyer requests in channels optimally and conflict-free is an NP-hard problem. Second, truthfulness is regarded as one of the most critical properties, which ensures that it is to the best interest of each buyer to bid with the true valuation that it deems for the requested spectrum resource. But designing a truthful spectrum auction mechanism is non-trivial when user requests have both spatial and temporal dimensions. Third, it is highly desirable for a practical spectrum auction mechanism to offer performance guarantee, which guards the seller's (or buyers') interest by ensuring

the auction result will be within a reasonable margin from the optimal result.

The prior art has only partially addressed the above challenges. There is an active line of research studying auction with spectrum spatial reuse [9], [11], [13], [17], [28], [29], [30], [38], [39], [40], [41], but they do not consider the temporal demands from buyers, where each buyer may only require a channel within a certain period of time and different buyers may have different time periods. While other work considers spectrum temporal reuse [8], [26], [31], they ignore spatial reuse by assuming that the conflict graph amongst buyers' geometry locations is a completed graph for each channel. Moreover, most of these auction mechanisms were designed to achieve truthfulness, without considering performance guarantee. Designing a truthful auction mechanism with provable performance guarantee is a harder problem, particularly if we want to support both spatial and temporal spectrum reuse. Solving this problem will require new techniques.

The auction model considered in this paper has a single seller and multiple buyers, with the latter bidding for channels that the former offers. We define a flexible optimization objective that can be set to either maximize the overall *social efficiency*, *i.e.*, allocating channels to buyers who *value* spectrum resource the most, or maximize the *expected revenue*, *i.e.*, allocating channels to buyers who will *pay* the most. Both are natural goals for spectrum auction.

Our model is different from double auction [9], [26], [28], [40], which involves multiple sellers and multiple buyers, with buyers bidding for resources and sellers bidding for demands. Research on double auction does not consider the maximization of revenue (or social efficiency) because it is not practically viable to maximize the overall revenue with multiple sellers each seeking its own interest. For example, although Feng et al. [9] considers spectrum reuse in both

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Manuscript received 1 Dec. 2013; revised 13 July 2014; accepted 22 July 2014.  
Date of publication 29 July 2014; date of current version 1 May 2015.  
For information on obtaining reprints of this article, please send e-mail to: reprints@ieee.org, and reference the Digital Object Identifier below.  
Digital Object Identifier no. 10.1109/TMC.2014.2343976

spatial and temporal domains, its double auction mechanism cannot be applied to our auction model because revenue (or social efficiency) optimality is not defined in double auction, let alone performance guarantee (which is based on a certain optimization target). Not surprisingly, the techniques used in their double auction are very different from ours. Also related is the work on combinatorial auction [8], [38], [42], where each buyer requests for a specific bundle of discrete resources, such as specific time blocks at specific channels [8]. This is different from our model (or the double auction model) where each buyer request may be allocated in one of multiple channels (or sellers) that meet the buyer's spectrum demand. In another related work [4], Chen and Zhong propose a spectrum auction framework for multiple collision domains, using a greedy-like channel allocation mechanism. However, this work mainly focuses on allocating spectrum with variable bandwidth to buyers, without providing a performance guarantee (which is the focus of our paper).

In this paper, we first design a strategy-proof spectrum auction framework, using an objective function based on virtual valuation, which can be flexibly turned into either social efficiency or revenue. With channels being reused spatially and temporally, we prove that it is an NP-hard problem to optimally allocate buyer requests in channels in order to maximize the social efficiency or the expected revenue. We develop an integer programming formulation for this optimal channel allocation problem, and relax it into a linear programming (LP) problem, which is solvable in polynomial time, resulting in a fractional solution for channel allocation. We then transform this fractional solution into a feasible integer solution of the original channel allocation problem by using a randomized rounding procedure that ensures the feasibility of the solution and good approximation to the objective function. We prove that the *expected* total valuation for the feasible integer solution is at least  $(1 - 1/e)$  times the total valuation of the optimal solution. However, the feasible solution produced by the randomized rounding procedure might be arbitrarily bad in the worst case. To achieve a performance guarantee, we propose a derandomized rounding algorithm, called DCA, to produce a feasible solution whose total valuation is *guaranteed* to be at least  $(1 - 1/e)$  times of the total valuation of the optimal solution.

On the other hand, truthfulness is a critical issue of spectrum auction; it guarantees the dominating strategy to bid the true valuation of the resource for each buyer. To this end, we propose a truthful auction mechanism called MDCA, which is built on top of DCA. It has been proved that an auction mechanism is truthful if its channel allocation algorithm is bid-monotone and the seller always charges a payment of the critical value from each winner [23]. Here, a bid-monotone channel allocation algorithm means that, once a buyer wins by bidding  $b_i$ , it will always win if it bids  $b'_i > b_i$ . The *critical value* is the minimum bid value for a buyer to win the auction. In this paper, we prove that the channel allocation mechanism of MDCA is bid-monotone. This implies that MDCA is a truthful auction mechanism.

Finally, in order to ensure that the critical values of user payment can be determined in polynomial time, we design a channel allocation and payment calculation mechanism, called CATE, which is another revised version of DCA and

has an approximation factor of  $(1 - 1/e)$ . We prove that CATE is truthful in expectation, which means that each buyer always maximizes its expected profit by revealing its true valuation. To the best of our knowledge, we are the first to design truthful spectrum auction mechanisms with performance guarantee with both spatial and temporal spectrum reuse.

The rest of the paper is organized as follows. We first present the spectrum auction model and the problem formulation in Section 2. Then, we develop an integer programming formulation for optimal channel allocation in this section. Our generic spectrum auction framework is discussed in the last of this section. In Sections 3, 4 and 5, we propose three spectrum auction mechanisms: DCA, MDCA, and CATE, respectively. To make our model more general, we also give a further discussion on how to relax some assumptions in Section 6. These spectrum auction mechanisms are evaluated through simulations in Section 7. We discuss the related literature in Section 8, and conclude the paper in Section 9.

## 2 PRELIMINARIES

### 2.1 Spectrum Auction Model

Auctions in our model are executed periodically. In each round, the spectrum owner (*a.k.a* seller) subleases the access right of  $m$  channels in certain fixed areas during time interval  $[0, T]$ ,<sup>1</sup> and  $n$  buyers request the usage of channels in fixed time intervals and geographical locations/areas. Our goal is to allocate these buyer requests in the channels, such that either social efficiency or revenue is maximized. We do not consider the problem of individual wireless users equipped with cognitive radio to dynamically acquire unused spectrum on the fly. The users in our model are more likely to be organizations or companies who need spectrum to support certain communication functions among their clients for a certain period of time. The spectrum allocation is performed offline beforehand.

Assume each channel provided by the seller has a set of conflict-free license areas and the seller only sells the right of accessing his under-used channels in the license areas. The license areas of different channels may be different, partially overlapped, or identical. We use  $\mathcal{S}$  to denote the set of channels, and define each channel  $s_j \in \mathcal{S}$  as  $s_j = (R_j, A_j)$ , where  $A_j$  is the set of license areas of channel  $s_j$ , and  $R_j$  is the interference radius of a transmission when a user transmits in channel  $s_j$ . For example, the license area set of channel  $s_4$  in Fig. 1 is  $A_4 = \{\text{Area 3, Area 4, Area 5}\}$ .

Let  $\mathcal{B}$  be the set of buyers, in which each buyer  $i \in \mathcal{B}$  has a request. We define buyer  $i = (v_i, r_i)$ , where  $v_i$  is buyer  $i$ 's true valuation which is not revealed to the seller, and  $r_i$  is the request of  $i$ . The auction mechanism we studied in this work is a sealed-bid offline auction. Each buyer only knows its own bid, and has no idea of others' bids. Moreover, each buyer can only submit its bid and request once in each round of auction. Since spectrum auction is actually a game between the seller and buyers, each buyer can also change

1. Note that our results apply to a more general model where, for each channel, the seller only subleases the access right for some time intervals in  $[0, T]$ .

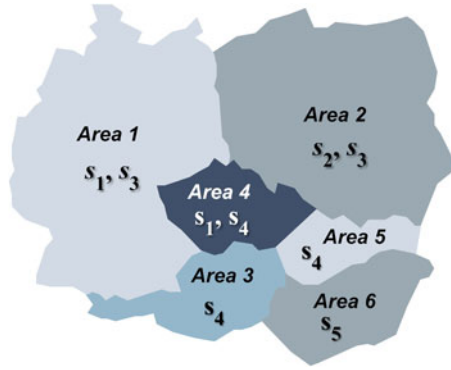


Fig. 1. The license areas of channels.

its strategy in the following auctions according to the history auction results. Let  $\mathcal{R}$  be the set of requests of buyers. To make our auction model more general, we consider two buyer request models. The first one is the *Point model*, in which each buyer requests for the usage of channel in a particular geographical location and during a fixed time interval. The second one is the *Area model*, in which each buyer requests for the usage of channel in a particular geographical area and during a fixed time interval. We assume that if a request is admitted, it can only be served by a unique channel. Buyers will not use one channel for certain time duration and then switch to another channel later for the same request. Moreover, the requested area of a buyer should be fully covered by the license areas of the allocated channel. For example, if a requested area is Area 2 or a portion of Area 2 in Fig. 1, this request can only be allocated in channel  $s_2$  or  $s_3$ .<sup>2</sup> Then, each request  $r_i \in \mathcal{R}$  can be defined as  $r_i = (L_i, b_i, a_i, t_i, d_i)$ , where  $L_i$  is buyer  $i$ 's geographical location in the *Point model* or the area where buyer  $i$  wants to access the channel in the *Area model*,  $b_i$  is the buyer's bidding price, and finally  $a_i$ ,  $d_i$ , and  $t_i$  denote the beginning time, the ending time, and the duration of channel usage, respectively. In this paper, we only consider the case of  $d_i - a_i = t_i$ , which means that the request from the buyer is a fixed time interval. We leave the case of  $d_i - a_i > t_i$  as future work. The beginning time and the ending time of the requests from different buyers can be different. The time lengths of different requests can also be different, either. Notice that in an auction, the value  $b_i$  provided by a buyer may be different from its true valuation  $v_i$ . When the dominating strategy for each user  $i$  is to bid  $b_i = v_i$ , we say the mechanism is truthful.

We say that two requests  $r_i$  and  $r_k$  conflict with each other if they satisfy the following constrains: (1) the distance between  $L_i$  and  $L_k$  is smaller than twice of the interference radius in the *Point model*, or  $L_i \cap L_k \neq \emptyset$  in the *Area model*; and (2) the required time intervals from  $r_i$  and  $r_k$  overlap with each other. We denote the conflict relationship among the requests by a conflict graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , where a vertex in  $\mathcal{V}$  represents a request in  $\mathcal{R}$ , and there is an edge  $(r_i, r_k) \in \mathcal{E}$  if requests  $r_i$  and  $r_k$  conflict with each other. Note that, for the same requests  $r_i$  and  $r_k$ , different interference radius in each channels will lead to a different conflict relationship.

2. We will give a further discussion on how to relax these assumptions in Section 6.

We use a matrix  $Y = (y_{i,k,j})_{n \times n \times m}$  to represent the conflict relationship in graph  $\mathcal{G}$ , where  $y_{i,k,j} = 1$  if requests  $r_i$  and  $r_k$  conflict with each other in channel  $j$ , or  $y_{i,k,j} = 0$  otherwise. Since the spectrum is a local resource, we need a location matrix  $C = (c_{i,j})_{n \times m}$  to represent whether  $L_i$  is in the license area of channel  $s_j$ , where  $c_{i,j} = 1$  if  $L_i$  is in the license regions of channel  $s_j$ , or  $c_{i,j} = 0$  otherwise. Therefore, two requests  $r_i$  and  $r_k$  can share channel  $s_j$  only if  $y_{i,k,j} = 0$ , and  $c_{i,j} = 1$ ,  $c_{k,j} = 1$ .

## 2.2 Problem Formulation

The objective of our work is to design a truthful (or strategy-proof) auction mechanism where the buyers send their concealed requests to the seller, who will then determine which requests are allocated in which channels as well as the payment from each buyer, such that the social efficiency or revenue is maximized. A mechanism is composed of two methods: *allocation method* and *payment computation method*. In a spectrum auction mechanism, the allocation method will determine which buyer will get which spectrum for what time intervals. The payment computation method will determine how much each buyer will pay for the allocated spectrum based on the bidding values from all buyers. A mechanism is said to be truthful or strategy-proof if a buyer bidding its truth bid will maximize its profit (i.e., the true valuation  $v_i$  minus its payment  $p_i$ ) regardless of the bid of other buyers. If we can find the allocation that maximizes the social efficiency, the payment by each buyer can be directly computed by the celebrated VCG mechanism. When considering different bids from other buyers, a buyer's profit under his/her own bid will be a random variable depending on other buyer's bid distribution. A mechanism is said to be *truthful in expectation* if for every buyer, bidding his/her true value will maximize the expected profit when considering the bid distributions of other buyers.

The payment from a buyer must be equal to or lower than the buyer's bidding price  $b_i$ , and it is determined in a way (explained shortly) that ensures a truthful auction. Since an auction mechanism is truthful if its channel allocation algorithm is bid-monotone and the seller always charges a payment of the critical value from each buyer, we divide the problem of designing the truthful spectrum auction mechanism into two parts: the bid-monotone channel allocation problem and the critical payment calculation problem, which are defined as follows.

*Bid-monotone channel allocation problem.* Given a request set  $\mathcal{R}$  and a channel set  $\mathcal{S}$ , the bid-monotone channel allocation problem is to determine which requests are allocated in which channels, so that the social efficiency or the revenue of seller is maximized, and that the channel allocation mechanism has the bid-monotone property. We say a channel allocation mechanism is bid-monotone, if and only if, when each winner  $i$  wins by bidding  $b_i$ , it will always win by bidding  $b'_i > b_i$ .

*Critical payment calculation problem.* Given the bid of all other buyers, the critical value of buyer  $i$  is such a bid value: when  $i$  bids no less than this value, it always win; otherwise, it will lose. To ensure the truthfulness of the auction mechanism, the critical payment calculation problem is to find the critical values for winners, given the bid of all other buyers.



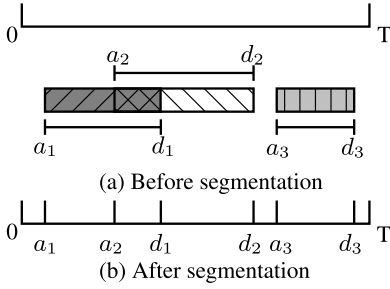


Fig. 2. An instance of the time interval segmentation.

Below we define our optimization objective.

*Social efficiency maximization.* The social efficiency for an auction  $\mathcal{M}$  is defined as  $\sum_{r_i \in \mathcal{R}} b_i x_i$ , where  $x_i = 1$  if buyer  $i$  wins in  $\mathcal{M}$ ; otherwise,  $x_i = 0$ . Then, the social efficiency maximization problem is to find a mechanism  $\mathcal{M}$  which can maximize the social efficiency.

*Revenue maximization.* The revenue of an auction is the total payment from the buyers. An auction that maximizes the revenue for the seller is known as an optimal auction in economic theory. Myerson introduces the notion of virtual valuation  $\phi_i(b_i)$  as

$$\phi_i(b_i) = b_i - \frac{1 - F_i(b_i)}{f_i(b_i)}, \quad (1)$$

where  $F_i$  is the probability distribution function for the true valuation of the spectrum resource requested by buyer  $i$ , and  $f_i(b_i) = \frac{dF_i(b_i)}{db_i}$  is the corresponding probability density function [22]. As in [17], we assume that the exact valuation of the requested resource at the present time is private information to the buyer, but its distribution  $F_i$  is known to the seller based on the records of history transactions.<sup>3</sup> According to the theory of optimal auction [22], maximizing the revenue is equivalent to finding the optimal solution that maximizes  $\sum_{r_i \in \mathcal{R}} \phi_i(b_i) x_i$ , where  $x_i = 1$  if buyer  $i$  wins in the auction; otherwise  $x_i = 0$ .

### 2.3 Optimal Channel Allocation

For channel allocation, we need to match requests and channels optimally under their constraints. For each request  $r_i$ , it can only be allocated in the time slice between  $a_i$  and  $d_i$ . For each channel  $s_j$ , it can only allocate time slices to the requests which are entirely in its license area. Moreover, we can only allocate channels to requests that are conflict-free of each other. In order to simplify the matching between requests and channels, we segment the available time of each channel into many time slices. Recall that the available time of each channel is  $[0, T]$  in each auction period. We use the arrival time  $a_i$  and deadline  $d_i$  of each request  $r_i$  to partition the time interval  $[0, T]$ . As shown in Fig. 2, the arrival times and deadlines of requests  $r_1$ ,  $r_2$  and  $r_3$  divide the time interval  $[0, T]$  into seven time slices. Suppose there are  $n$  requests, it is easy to see that the time interval  $[0, T]$  will be divided into no more than  $2n + 1$  time slices. Next, we will formulate the channel allocation problem.

3. With a truthful auction mechanism, to the best interest of each buyer  $i$ , the bidding price  $b_i$  is expected to be the true valuation of the spectrum resource at the time of the request.

First, the time slices of channel  $s_j$  can only be allocated to the requests within the license area of the channel. Let  $x_{i,j}^l$  be an indicator variable for whether the  $l$ th time slice of channel  $s_j$  is allocated to request  $r_i$ . We have the constraint  $x_{i,j}^l \leq c_{i,j}$ .

Second, each time slice can only be allocated to requests that are conflict-free of each other. Thus, we have another constraint  $\sum_{k \neq i} x_{k,j}^l y_{i,k,j} + x_{i,j}^l \leq 1$ .

Third, let  $t_j^l$  be the length of  $l$ th time slice in channel  $s_j$ . With a little abuse of notation, we use  $a_i$  to denote the first time slice that  $r_i$  wants to use, and  $d_i$  the last time slice that  $r_i$  wants to use. If we allocate request  $r_i$  in channel  $s_j$ , the time assigned to request  $r_i$  from channel  $s_j$  should be equal to the required time of request  $r_i$ . Hence, we have  $\sum_{l=a_i}^{d_i} x_{i,j}^l t_j^l = t_i x_{i,j}$ .

From the analysis above, the allocation problem can be formulated as an integer programming IP(1) below.

$$\max \sum_{s_j \in \mathcal{S}} \sum_{r_i \in \mathcal{R}'} \phi_i(b_i) x_{i,j},$$

subject to

$$\begin{cases} \sum_{s_j \in \mathcal{S}} x_{i,j} \leq 1, \forall r_i \in \mathcal{R}' \\ x_{i,j}^l \leq c_{i,j}, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}', \forall l \\ \sum_{k \neq i} x_{k,j}^l y_{i,k,j} + x_{i,j}^l \leq 1, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}', \forall l \\ \sum_{l=a_i}^{d_i} x_{i,j}^l t_j^l = t_i x_{i,j}, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}' \\ x_{i,j} \in \{0, 1\}, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}' \\ x_{i,j}^l \in \{0, 1\}, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}', \forall l, \end{cases}$$

where  $x_{i,j}$  indicates whether channel  $s_j$  is allocated to request  $r_i$ ,  $y_{i,k,j}$  indicates whether request  $r_i$  conflicts with request  $r_k$  in channel  $s_j$ , and  $\sum_{s_j \in \mathcal{S}} \sum_{r_i \in \mathcal{R}'} \phi_i(b_i) x_{i,j}$  is called the *total valuation*, which is to be maximized.

If we obtain the optimal solution of the integer programming IP(1), we can design a truthful auction mechanism by applying the well-known Vickrey-Clarke-Groves (VCG) mechanism. Unfortunately, the optimal channel allocation problem is NP-hard. Thus, VCG mechanism can be used only if the optimal allocation can be computed using non-polynomial time method for the problem with small input size. Otherwise, we cannot apply the VCG mechanism to compute the payment in a truthful mechanism based on an approximation allocation method. In order to tackle the NP-hardness, we need to design approximation allocation method, implying that the celebrated VCG mechanisms cannot be applied here. In the following sections, we will employ the LP relaxation method to design a series of polynomial-time channel allocation mechanisms with an approximation factor of  $1 - 1/e$ .

**Theorem 1.** *The optimal channel allocation problem is NP-hard.*

**Proof.** Consider a simple case where there is only one channel. Given a conflict graph  $\mathcal{G}$ , we need to allocate requests in this channel. This simple case of channel allocation is equivalent to finding maximum weighted independent sets, which is an NP-hard problem.  $\square$

## 2.4 A Truthful Spectrum Auction Framework

In this section, we propose a general truthful spectrum auction framework with the goal of maximizing social efficiency or revenue, as shown in Algorithm 1. In our framework, we can flexibly choose different optimization targets according to the practical requirements of auction problems. The details are described below.

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### Algorithm 1. Spectrum Auction Framework

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**Input:** the set of channels  $\mathcal{S}$ , the set of requests  $\mathcal{R}$ , and the monotone allocation and payment mechanism  $\mathcal{A}$ ;  
**Output:** the channel allocation vector  $\mathbf{X}$ , and the payment vector  $\mathbf{P}$ ;

- 1:  $\mathcal{R}' = \mathcal{R}$ ;
- 2: **for** each  $r_i \in \mathcal{R}$  **do**
- 3:      $p_i = 0$ ;
- 4:     **if** optimization target is to maximize social efficiency **then**
- 5:          $\phi_i(b_i) = b_i$ ;
- 6:     **else**
- 7:          $\phi_i(b_i) = b_i - \frac{1-F_i(b_i)}{f_i(b_i)}$ ;
- 8:         **if**  $\phi_i(b_i) < \eta^\phi t_i$  **then**
- 9:              $\mathcal{R}' = \mathcal{R}'/r_i$ ;
- 10:     Run  $\mathcal{A}$  using the set of virtual valuations  $\{\phi_i(b_i)\}_{r_i \in \mathcal{R}'}$ ;
- 11:     Let  $\mathbf{X} = (x_i)_{r_i \in \mathcal{R}'}$  be the channel allocation and  $\tilde{\mathbf{P}} = (\tilde{p}_i)_{r_i \in \mathcal{R}'}$  be the corresponding payment returned by  $\mathcal{A}$ ;
- 12:     **for** each  $x_i = 1$  **do**
- 13:         **if** target is to maximize social efficiency **then**
- 14:              $p_i = \tilde{p}_i$ ;
- 15:         **else**
- 16:              $p_i = \phi_i^{-1}(\tilde{p}_i)$ ;
- 17:     **return**  $(\mathbf{X}, \mathbf{P})$ ;

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At the beginning of every auction period, we choose an optimization target. If we choose the social efficiency maximization as our target, we let the virtual valuation  $\phi_i(b_i) = b_i$ . Then, we use the set  $\Phi = (\phi_i(b_i))_{r_i \in \mathcal{R}'}$  as input to the channel allocation and payment calculation mechanism  $\mathcal{A}$ .  $\mathcal{A}$  returns an optimal channel allocation  $\mathbf{X} = (x_i)_{r_i \in \mathcal{R}'}$ , which maximizes  $\sum_{r_i \in \mathcal{R}} \phi_i(b_i)x_i$ , where  $x_i = 1$  means that buyer  $i$  wins the auction, and  $x_i = 0$  means it loses. Meanwhile,  $\mathcal{A}$  also returns a corresponding payment vector  $\tilde{\mathbf{P}} = (\tilde{p}_i)_{r_i \in \mathcal{R}'}$ , and we charge each buyer  $p_i = \tilde{p}_i$ .

If we choose to maximize the revenue of the seller, we convert the bid of each buyer into its corresponding virtual valuation by setting  $\phi_i(b_i) = b_i - \frac{1-F_i(b_i)}{f_i(b_i)}$ . Then, we can use the same allocation mechanism  $\mathcal{A}$  as in the case of social efficiency to maximize  $\sum_{r_i \in \mathcal{R}} \phi_i(b_i)x_i$ . To ensure the worst case profit, the seller may set a virtual reservation price  $\eta^\phi$ , which is the minimum price for spectrum usage *per unit time*. We remove the requests  $r_i$  whose virtual valuations are smaller than  $\eta^\phi t_i$ , and use the remaining requests as input of  $\mathcal{A}$ , which returns an allocation vector  $\mathbf{X}$  and the corresponding payment vector  $\tilde{\mathbf{P}}$ . Different from the previous optimization target, the payment vector  $\tilde{\mathbf{P}}$  we obtain in this case contains virtual payments of the buyers. Therefore, we need to convert the virtual payments back into the actual payments through  $p_i = \phi_i^{-1}(\tilde{p}_i)$ .

As we have discussed previously, if mechanism  $\mathcal{A}$  is a monotonic allocation and it always charges each winning buyer its critical value, the proposed auction framework is truthful. To this end, we give our solution on designing the bid-monotone channel allocation mechanisms and calculating the critical value for each winner in the following three sections.

## 3 (1-1/e)-APPROXIMATION CHANNEL ALLOCATION METHODS

The LP relaxation technique can often be used to design a good approximation algorithm for NP-hard problems. In this section, we present a randomized method for channel allocation problem by using LP relaxation technique. We relax IP(1) to linear programming LP(2) by replacing  $x_{i,j} \in \{0, 1\}$  with  $0 \leq x_{i,j} \leq 1$ , and by replacing  $x_{i,j}^l \in \{0, 1\}$  with  $0 \leq x_{i,j}^l \leq 1$ . Then,  $x_i = \sum_{s_j \in \mathcal{S}} x_{i,j}$ . The allocation problem is reformulated as the following relaxed LP problem:

$$\max \sum_{s_j \in \mathcal{S}} \sum_{r_i \in \mathcal{R}'} \phi_i(b_i)x_{i,j},$$

subject to

$$\begin{cases} \sum_{s_j \in \mathcal{S}} x_{i,j} \leq 1, \forall r_i \in \mathcal{R}' \\ x_{i,j}^l \leq c_{i,j}, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}', \forall l \\ \sum_{k \neq i} x_{k,j}^l y_{i,k,j} + x_{i,j}^l \leq 1, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}', \forall l \\ \sum_{l=a_i}^{d_i} x_{i,j}^l t_j^l = t_i x_{i,j}, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}' \\ x_{i,j}^l = x_{i,j}^{l'}, \forall r_i \in \mathcal{R}', \forall l, l' \in [a_i, d_i] \\ 0 \leq x_{i,j} \leq 1, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}' \\ 0 \leq x_{i,j}^l \leq 1, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}', \forall l. \end{cases}$$

Recall that the number of time slices is no more than  $2n + 1$  for each channel, so LP(2) has a polynomial number of variables and constraints, and can be solved optimally in polynomial time. Clearly, the solution of this relaxed LP formulation is often not feasible. Then we need to convert the solution of LP(2) to a feasible solution for IP(1), the channel allocation problem.

### 3.1 Randomized Rounding

Let  $O_{LP2}$  be the optimal solution of LP(2). We apply the standard randomized rounding to obtain a feasible integer solution  $f_{IP1}$  to IP(1). The rounding procedure is presented as follows:

- Randomly choose a channel  $s_j$ , for any request with  $x_{i,j} > 0$ , choose  $r_i$  for channel  $s_j$  with probability  $x_{i,j}$ , and if chosen, set  $x_{i,j} = 1$ ;
- If  $x_{i,j} = 1$ , set  $x_{k,j} = 0$  for all requests  $r_k$  with  $y_{i,k,j} = 1$ ;
- If  $x_{i,j} = 1$ , set  $x_{i,k} = 0$  for all channels with  $k \neq j$ .
- Repeat steps 1 to 3 until all requests have been processed.

Through the randomized rounding procedure above, the optimal solution of LP(2) is converted into a feasible solution of IP(1). Let  $w_{O_{LP2}}$  be the total valuation of  $O_{LP2}$ ,

and let  $E(w_{f_{IP1}})$  be the expected total valuation of  $f_{IP1}$ . We show by Theorem 2 that  $E(w_{f_{IP1}}) \geq (1 - 1/e)w_{OLP2}$ .

**Theorem 2.** *The expected total valuation of the rounded solution is at least  $(1 - 1/e)$  times the total valuation of the optimal solution to LP(2).*

**Proof.** For each request  $r_i$ , let  $H = \{s_j \in \mathcal{S} \mid x_{i,j} > 0\}$  be the set of channels  $s_j \in \mathcal{S}$  with  $x_{i,j} > 0$ , and let  $h = |H|$ . Clearly,  $0 \leq h \leq m$ . The probability that request  $r_i$  is not allocated in any channel by  $f_{IP1}$  is  $\prod_{j=1}^h (1 - x_{i,j})$ . Let  $q_i$  denote the probability that request  $r_i$  is allocated in one of the  $h$  channels by  $f_{IP1}$ . Then, we have  $q_i = 1 - \prod_{j=1}^h (1 - x_{i,j})$ . It's obvious that  $E(w_{f_{IP1}}) = w_{OLP2}$  when  $h = 0$  or  $1$ . Thus, we only consider the case  $h \geq 2$  in the following. In this case,  $q_i$  is minimized when  $x_{i,j} = x_i/h$ . Hence, we have  $q_i \geq 1 - (1 - x_i/h)^h$ , and

$$\frac{q_i}{x_i} \geq \frac{1}{x_i} (1 - (1 - (x_i/h)^h)). \quad (2)$$

The right side of the inequality is a monotonically decreasing function of  $x_i$ , with  $0 \leq x_i \leq 1$ . Thus, it is minimized when  $x_i = 1$ , and we have

$$\begin{aligned} \frac{q_i}{x_i} &\geq (1 - (1 - 1/h)^h) \\ &\geq 1 - \frac{1}{e} + \frac{1}{32h^2} \\ &\geq 1 - 1/e. \end{aligned} \quad (3)$$

For each request  $r_i$  with  $q_i > 0$ , its contribution to the expected total valuation of the rounded solution is  $q_i \phi_i(b_i)$ , and its contribution to the total valuation of the optimal solution of LP(2) is  $x_i \phi_i(b_i)$ . Hence we have  $\frac{q_i \phi_i(b_i)}{x_i \phi_i(b_i)} \geq 1 - \frac{1}{e}$ . Since this inequality holds for any request  $r_i \in \mathcal{R}'$ , and  $E(w_{f_{IP1}}) = \sum_{r_i \in \mathcal{R}'} q_i \phi_i(b_i)$ ,  $w_{OLP2} = \sum_{r_i \in \mathcal{R}'} x_i \phi_i(b_i)$ , we must have  $E(w_{f_{IP1}}) \geq (1 - 1/e)w_{OLP2}$ .  $\square$

We have shown that the expected total valuation of feasible solution  $f_{IP1}$  to IP(1) obtained by our randomized rounding is at least  $(1 - 1/e)$  times the total valuation of the optimal solution to LP(2). Obviously, the total valuation of the optimal solution to LP(2), which is denoted by  $w_{OLP2}$ , is no less than that of the optimal solution to IP(1), which is denoted by  $w_{OIP1}$ . Therefore, we have the following theorem.

**Theorem 3.** *The expected total valuation of the rounded solution is at least  $(1 - 1/e)$  times the total valuation of the optimal solution to IP(1).*

The random rounding procedure just ensures that the expected total valuation of  $f_{IP1}$  is at least  $(1 - 1/e)$  times of the total valuation of  $O_{LP2}$ . Consider a simple case, in which there are only one channel and two requests. We use  $r_1$  and  $r_2$  to denote the two requests. Suppose  $t_1 \gg t_2$  and  $b_1 \gg b_2$ , the per-unit bid of  $r_1$  is smaller than that of  $r_2$ , and the requested time slots of  $r_1$  and  $r_2$  overlap with each other. In this case,  $x_2$  may not equal to zero in the optimal solution of LP(2). Thus, our random rounding procedure may allocate  $r_2$  in the channel. Since the bid of  $r_2$  can be arbitrarily close to zero, the feasible solution produced by the random rounding procedure might be arbitrarily bad.

### 3.2 Derandomized Rounding

To achieve a performance guarantee in the worst case, we need to find a feasible solution of IP(1) whose total valuation is *always* no less than  $(1 - 1/e)$  times of  $w_{OLP2}$ . In the following, we show that this can be achieved through a derandomized rounding procedure.

Let  $E(w_{f_{IP1}} \mid r_i \rightarrow s_j)$  be the expected total valuation when request  $r_i$  is allocated in channel  $s_j$ , and  $E(w_{f_{IP1}} \mid \tilde{i})$  be the expected total valuation when request  $r_i$  is not allocated in any channel.

$$E(w_{f_{IP1}}) = \sum_{r_j \in \mathcal{S}} E(w_{f_{IP1}} \mid r_i \rightarrow s_j) q_{i,j} + E(w_{f_{IP1}} \mid \tilde{i}) \tilde{q}_i, \quad (4)$$

where  $q_{i,j}$  denotes the probability that request  $r_i$  is allocated in channel  $s_j$ , and  $\tilde{q}_i$  denotes the probability that  $r_i$  is not allocated in any channel. Hence, there must exist at least one conditional expectation in  $E(w_{f_{IP1}} \mid r_i \rightarrow s_1), \dots, E(w_{f_{IP1}} \mid r_i \rightarrow s_m), E(w_{f_{IP1}} \mid \tilde{i})$ , which is larger than or equal to  $E(w_{f_{IP1}})$ .

Next, we show how to compute  $E(w_{f_{IP1}})$  and  $E(w_{f_{IP1}} \mid r_i \rightarrow s_j), \forall s_j \in \mathcal{S}$ . First, we know that

$$E(w_{f_{IP1}}) = \sum_{r_i \in \mathcal{R}'} \phi_i(b_i) q_i, \quad (5)$$

where  $q_i$  is the probability that request  $r_i$  is allocated in one of the channels, and it can be computed by

$$q_i = 1 - \prod_{s_j \in \mathcal{S}} (1 - x_{i,j}). \quad (6)$$

Next, let  $q_{r_i \rightarrow s_j, k}$  be the probability that request  $r_k$  is allocated in a channel when request  $r_i$  is allocated in  $s_j$ . It can be calculated by

$$q_{r_i \rightarrow s_j, k} = \begin{cases} 1 - \prod_{o \neq j} (1 - x_{k,o}), & \text{if } y_{i,k,j} = 1; \\ q_k, & \text{otherwise.} \end{cases} \quad (7)$$

Hence, we can compute  $E(w_{f_{IP1}} \mid r_i \rightarrow s_j)$  as follows:

$$E(w_{f_{IP1}} \mid r_i \rightarrow s_j) = \phi_i(b_i) + \sum_{k \neq i} \phi_k(b_k) q_{r_i \rightarrow s_j, k}. \quad (8)$$

We now describe the derandomized channel allocation (DCA) procedure. We sort all requests by their arrival times  $a_i$  in ascending order to decide which request should be allocated in which channel. Let  $x_i = \sum_{j \in \mathcal{S}} x_{i,j}$ . For any request with  $x_i = 0$ , we reject the request. Without loss of generality, let  $r_i$  be the first request with  $x_i > 0$  in the order. We examine  $E(w_{f_{IP1}} \mid r_i \rightarrow s_j)$  for all  $s_j \in \mathcal{S}$  to see if any of them satisfies  $E(w_{f_{IP1}} \mid r_i \rightarrow s_j) \geq E(w_{f_{IP1}})$ .

- If we find one, we allocate request  $r_i$  in channel  $s_j$ , and perform the following operations: (1) set  $x_{i,j} = 1$  and  $x_i = 1$ , (2) set  $x_{i,k} = 0$  for all channels with  $k \neq j$ , and (3) set  $x_{k,j} = 0$  for all requests  $r_k$  with  $y_{i,k,j} = 1$ .
- If we do not find one,  $E(w_{f_{IP1}} \mid \tilde{i}) \geq E(w_{f_{IP1}})$  must hold, and we reject request  $r_i$ .

After request  $r_i$  is processed, we have

$$E(w_{f_{IP1}} | cond1) \geq E(w_{f_{IP1}}), \quad (9)$$

where  $cond1$  is either  $r_i \rightarrow s_j$  or  $r_i$  is rejected, depending on which of the above two cases has been performed.

We repeat the above process on other requests one after another to determine whether it is allocated in a channel and if so, which channel. More specifically, after we process the next request  $r_{i'}$  after  $r_i$  using the same process except for replacing  $f_{IP1}$  with  $f_{IP1} | cond1$ , we will have

$$E(w_{f_{IP1}} | cond1, cond2) \geq E(w_{f_{IP1}} | cond1), \quad (10)$$

where  $cond2$  is either  $r_{i'} \rightarrow s_{j'}$  or  $r_{i'}$  is rejected.

Furthermore, after we process yet the next request  $r_{i''}$  with  $f_{IP1} | cond1$  replaced by  $f_{IP1} | cond1, cond2$ , we will have

$$E(w_{f_{IP1}} | cond1, cond2, cond3) \geq E(w_{f_{IP1}} | cond1, cond2), \quad (11)$$

where  $cond3$  is either  $r_{i''} \rightarrow s_{j''}$  or  $r_{i''}$  is rejected.

We keep an invariant that the conditional expectation  $E(w_{f_{IP1}} | \dots)$  never decreases. After processing all the requests, we will have a feasible solution of IP(1), whose total valuation is larger than or equal to  $E(w_{f_{IP1}})$ , i.e., at least  $(1 - 1/e)w_{OLP2}$ , according to the transitive inequalities of (9), (10), (11), ...

The pseudo code of the above derandomized channel allocation algorithm is given in Algorithm 2. Furthermore, we have the following theorem.

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**Algorithm 2.** DCA: Derandomized Channel Allocation

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**Input:** the set of channels  $\mathcal{S}$ , and the set  $\mathcal{R}'$  (which are sorted in an ascending order of  $a_i$ );  
**Output:** the channel allocation vector  $\mathbf{X}^*$ ;

- 1: Solve LP(2) optimally;
- 2: **for**  $i = 1$  to  $n$  **do**
- 3:     **if**  $x_i > 0$  **then**
- 4:         **for**  $j = 1$  to  $m$  **do**
- 5:             **if**  $E(w_{f_{IP1}}) \leq E(w_{f_{IP1}} | r_i \rightarrow s_j)$  **then**
- 6:                 set  $x_{i,j} = 1, x_i = 1$ ;
- 7:                 set all  $x_{i,k} = 0$  and  $x_{i,k}^l = 0$  if  $k \neq j$ ;
- 8:                 set all  $x_{k,j} = 0$  and  $x_{k,j}^l = 0$  if  $k \neq i$  and  $y_{i,k,j} = 1$ ;
- 9:             modify  $f_{IP1}$  by assigning  $r_i$  in  $s_j$  and update  $\mathbf{X}^*$ ;
- 10:             **break**
- 11:     **if**  $x_i \neq 1$  **then**
- 12:          $x_i = 0$ ;
- 13:     modify  $f_{IP1}$  by rejecting  $r_i$  and update  $\mathbf{X}^*$ ;
- 14: **return**  $\mathbf{X}^*$ ;

---

**Theorem 4.** DCA can be executed in polynomial time.

**Proof.** As mentioned above, LP(2) can be solved in polynomial time. Then, we allocate requests in channels with time complexity  $O(nm)$  in DCA with the optimal solution of LP(2). This finishes the proof.  $\square$

In our spectrum auction model, we assume that one request can only be allocated in one unique channel.

However, we relax IP(1) to a linear programming LP(2) by replacing  $x_{i,j} \in \{0, 1\}$  with  $0 \leq x_{i,j} \leq 1$ , and by replacing  $x_{i,j}^l \in \{0, 1\}$  with  $0 \leq x_{i,j}^l \leq 1$ . In LP(2), we only restrict that the time assigned to a request  $r_i$  from all the channels should be no longer than the required time of request  $r_i$ . In other words, a request can be served by different channels in LP(2). We prove that the total valuation of the solution of DCA is at least  $(1 - 1/e)$  times of the total valuation of the optimal solution of LP(2). Thus, DCA can also achieve the approximation factor of  $(1 - 1/e)$ , even if we release the assumption of one request being allocated in one unique channel.

## 4 A TRUTHFUL SPECTRUM AUCTION MECHANISM

Recall that to ensure the truthfulness of our auction mechanism, the allocation algorithm must be bid-monotone. This means that if request  $r_i$  wins the auction with bid  $b_i$ , it always wins with bid  $b_i' > b_i$ . In Algorithm 2, request  $r_i$  wins in the auction only if there exists a channel  $s_j$  which satisfies  $E(w_{f_{IP1}} | r_i \rightarrow s_j) \geq E(w_{f_{IP1}})$ . However, with the arbitrary selection of a channel that satisfies this condition and the use of (8), we find that it is hard to judge if  $E(w_{f_{IP1}} | r_i \rightarrow s_j)$  is still larger than  $E(w_{f_{IP1}})$  when request  $r_i$  increases its bid. We cannot prove or disprove the bid-monotone property of the allocation method DCA. Thus, it is unknown whether we can design a truthful mechanism based on this method. In the following, we revise DCA and show that the revised method does satisfy the bid-monotone property.

The first revision is to find the channel that has the largest conditional expectation  $\max_{s_j \in \mathcal{S}} E(w_{f_{IP1}} | r_i \rightarrow s_j)$ . If we allocate  $r_i$  in the channel with the maximal conditional expectation as long as  $\max_{s_j \in \mathcal{S}} E(w_{f_{IP1}} | r_i \rightarrow s_j) \geq E(w_{f_{IP1}} | \hat{i})$ , and do not allocate  $r_i$  in any channel otherwise, we can obviously obtain a feasible solution of IP(1), whose weight is as good as  $E(w_{f_{IP1}})$ , because either  $\max_{s_j \in \mathcal{S}} E(w_{f_{IP1}} | r_i \rightarrow s_j)$  or  $E(w_{f_{IP1}} | \hat{i})$  must be larger than or equal to  $E(w_{f_{IP1}})$ .

The second revision is an improved definition of  $E(w_{f_{IP1}} | r_i \rightarrow s_j)$  and  $E(w_{f_{IP1}} | \hat{i})$  as follows:

$$E(w_{f_{IP1}} | r_i \rightarrow s_j) = \phi_i(b_i) + E_{k \neq i}(w_{f_{IP1}} | r_i \rightarrow s_j), \quad (12)$$

where  $E_{k \neq i}(w_{f_{IP1}} | r_i \rightarrow s_j)$  is the expected weight of all other requests when request  $r_i$  has been allocated in channel  $s_j$ . We can compute it by allocating  $r_i$  in channel  $s_j$  first, and then solve LP(2) optimally with other requests. Because optimization is performed after allocating  $r_i$  in  $s_j$ , (12) produces an equal or higher expected value than (8) at the cost of additional computation overhead for solving LP(2).

$$E(w_{f_{IP1}} | \hat{i}) = E_{\mathcal{R}'/r_i}(w_{f_{IP1}}), \quad (13)$$

where  $E_{\mathcal{R}'/r_i}(w_{f_{IP1}})$  is the expected weight of all other requests when request  $r_i$  does not be allocated in any channel. We can compute it by solving LP (2) optimally with requests except  $r_i$ .

We give the revised version of Algorithm DCA as follows.

In MDCA, we first sort all of the requests by their arrival times in ascending order, and then scan all requests one by



one to decide which request can be allocated in channels. When request  $r_i$  is considered, we compute  $E(w_{f_{IP1}} | r_i \rightarrow s_j)$  for all channels  $s_j \in \mathcal{S}$  that no request conflicting with it has been allocated in. We allocate  $r_i$  in channel  $s_k$  when  $E(w_{f_{IP1}} | r_i \rightarrow s_k) = \max_{s_j \in \mathcal{S}} E(w_{f_{IP1}} | r_i \rightarrow s_j) \geq E(w_{f_{IP1}} | \tilde{i})$ , and reject it otherwise; note that in order to simplify notations we do not include the conditions for channel allocation of requests prior to  $r_i$ . After the last request was considered in MDCA, we get a feasible solution of IP(1), whose weight is as good as  $E(w_{f_{IP1}})$ .

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**Algorithm 3.** MDCA: Monotone Derandomized Channel Allocation Based on Linear Programming

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**Input:** the set of channels  $\mathcal{S}$ , and the set  $\mathcal{R}'$  (which are sorted in an ascending order of  $a_i$ );  
**Output:** the channel assignment  $\mathbf{X}^*$  ;

- 1: Solve LP(2) optimally;
- 2: **for**  $i = 1$  to  $n$  **do**
- 3:   **for**  $j = 1$  to  $m$  **do**
- 4:    **if**  $x_{i,j} > 0$  **then**
- 5:       $E(w_{f_{IP1}} | r_i \rightarrow s_k) = \max_{s_j \in \mathcal{S}} E(w_{f_{IP1}} | r_i \rightarrow s_j)$
- 6:      **if**  $E(w_{f_{IP1}} | r_i \rightarrow s_k) \geq E(w_{f_{IP1}} | \tilde{i})$  **then**
- 7:        set  $x_{i,j} = 1, x_i = 1$ ;
- 8:        set all  $x_{i,k} = 0$  and  $x_{i,k}^1 = 0$  if  $k \neq j$ ;
- 9:        set all  $x_{k,j} = 0$  and  $x_{k,j}^1 = 0$  if  $k \neq i$  and  $y_{i,k,j} = 1$ ;
- 10:      Break
- 11: **if**  $x_i \neq 1$  **then**
- 12:     $x_i = 0$ ;
- 13: **return**  $\mathbf{X}^*$ ;

---

**Theorem 5.** MDCA (see Algorithm 3) is bid monotone.

**Proof.** Suppose request  $r_i$  wins the auction with the bid  $b_i$ , and it is allocated with the channel  $s_j$ . To prove by contradiction, we assume that  $r_i$  cannot be allocated in any channel with the bid  $b'_i > b_i$ . There are two possible cases.

*Case 1.*  $\max_{s_j \in \mathcal{S}} E(w_{f_{IP1}} | r_i \rightarrow s_j) < E(w_{f_{IP1}} | \tilde{i})$  when  $r_i$  bids some value  $b'_i$  that is greater than  $b_i$ . However, when  $r_i$  increases its bid, clearly  $E_{k \neq i}(w_{f_{IP1}} | r_i \rightarrow s_j)$  and  $E(w_{f_{IP1}} | \tilde{i})$  keep invariant, and  $E(w_{f_{IP1}} | r_i \rightarrow s_j) \geq E(w_{f_{IP1}} | \tilde{i})$  always holds in this case. Thus, our hypothesis does not hold in this case.

*Case 2.* When  $r_i$  is considered with bid  $b'_i > b_i$ , the channel  $s_j$  has been occupied by a previously processed request  $r_l$ , which conflicts with  $r_i$ . Obviously,  $r_l$  is not allocated in  $s_j$  when  $r_i$  bids  $b_i$ . That means  $E(w_{f_{IP1}} | r_l \rightarrow s_j) < E(w_{f_{IP1}} | \tilde{l})$  or the channel  $s_j$  has been occupied by other requests which conflict with  $r_l$  but conflict-free with  $r_i$  when  $r_l$  was considered. In the first subcase, the contribution of  $r_i$  in  $E(w_{f_{IP1}} | \tilde{l})$  is larger than the contribution in  $E(w_{f_{IP1}} | r_l \rightarrow s_j)$ . Then, the increment of  $E(w_{f_{IP1}} | \tilde{l})$  is larger than that of  $E(w_{f_{IP1}} | r_l \rightarrow s_j)$  when  $r_i$  increases its bid from  $b_i$  to  $b'_i$ . Thus,  $r_i$  cannot be allocated in  $s_j$  when  $r_i$  bids  $b'_i > b_i$ . Assume that  $r_k$  which conflicts with  $r_l$  is allocated in  $s_j$  when  $r_i$  bids  $b'_i$  in the second subcase. However, the contribution of  $r_i$  in  $E(w_{f_{IP1}} | r_k \rightarrow s_j)$  is no less than the contribution in  $E(w_{f_{IP1}} | \tilde{k})$ . Thus, the increment of  $E(w_{f_{IP1}} | r_k \rightarrow s_j)$  is larger than that of

$E(w_{f_{IP1}} | \tilde{k})$  when  $r_i$  increases its bid from  $b_i$  to  $b'_i$ .  $r_k$  will also be allocated in  $s_j$  when  $r_i$  bids  $b'_i > b_i$ . In conclusion,  $r_i$  cannot be allocated in  $s_j$  when  $r_i$  bids  $b'_i > b_i$ .

Based on the analysis above, if  $r_i$  wins the auction with a bid  $b_i$ , it always wins with the bid  $b'_i > b_i$ .  $\square$

**Theorem 6.** MDCA can be executed in polynomial time.

**Proof.** We have shown that LP(2) can be solved in polynomial time. For each request with  $x_i \geq 0$  in the optimal solution of LP(2), we solve LP(2) no more than  $m$  times to check if request  $r_i$  can be allocated in a channel. Hence, the time complexity of MDCA is  $O(nm)$  multiplied by the time complexity of solving LP(2). This completes the proof.  $\square$

**Theorem 7.** We can build a truthful spectrum auction mechanism based on MDCA.

**Proof.** We have proved that MDCA is bid-monotone and it can be executed within a polynomial time. To build a truthful auction mechanism, we need to charge each winner its critical value. Fortunately, we can find the critical value for each winner by using a binary search method. Thus, we can build a truthful spectrum auction mechanism based on MDCA.  $\square$

We have shown that buyers will truthfully report their valuations in MDCA. Now, we prove that buyers cannot benefit from misreporting their requested time slots and areas either.

**Theorem 8.** Buyers will report their requested time slots and areas truthfully in MDCA.

**Proof.** As the buyers will not report a smaller requested area or less time slots than what they need, we only need to consider the case that each buyer may request a larger area or more time slots than those he needs. Actually, when a request  $r_i$  submits a larger area or more time slots,  $E(w_{f_{IP1}} | \tilde{i})$  will keep unchanged and  $\max_{s_j \in \mathcal{S}} E(w_{f_{IP1}} | r_i \rightarrow s_j)$  will decrease. Thus,  $r_i$  will still lose by misreporting if he loses by bidding truthfully. Note that the payment of each winner is his critical value. When  $r_i$  submits a larger area or more time slots, there will be some additional requests which overlap with  $r_i$ . As a result, the critical value of  $r_i$  will increase. Thus,  $r_i$  cannot benefit from misreporting his need. Since each buyer is rational and selfish, he will submit his request truthfully. This completes the proof.  $\square$

However, binary search can be slow, depending on the ratio of the max bid among requests to the step size of bids. To address this issue, we further design another channel allocation mechanism that is efficient in determining critical values and truthful in expectation.

## 5 A TRUTHFUL IN EXPECTATION SPECTRUM AUCTION MECHANISM

Although we fail to prove the truthfulness of DCA, we can revise DCA to derive a truthful spectrum auction mechanism in expectation, called CATE. It is more efficient than MDCA, and it also has an approximation factor of  $1 - 1/e$ .



The basic idea is depicted as follows. With the optimal solution of LP(2),  $\mathbf{X} = (x_i)_n$ , we first employ the technique proposed by Lavi and Swamy [18] to obtain a set of feasible solutions of IP(1),  $\mathcal{L}$ , by allocating requests with  $x_i \geq 0$  in channels. The size of  $\mathcal{L}$  can be made polynomial; see the proof of Theorem 13. Each feasible solution  $f \in \mathcal{L}$  has a probability  $q(f)$  of being chosen as the final solution. We will explain how to compute  $q(f)$  shortly. Let  $\mathbf{X}^f = \{x_i^f\}_{r_i \in \mathcal{R}}$  denote the channel allocation vector of a feasible solution  $f \in \mathcal{L}$ . Let  $x_i^f = 1$  denote that request  $r_i$  wins in solution  $f$ , and  $x_i^f = 0$  denote that  $r_i$  loses. The overall probability for request  $r_i$  to win in the final chosen solution is  $\sum_{f \in \mathcal{L}} x_i^f q(f)$ . Let  $\sum_{f \in \mathcal{L}} x_i^f q(f) = \frac{x_i}{\alpha}$ . We want to establish  $\alpha = \frac{e}{e-1}$ , so that the expected total valuation of CATE will be  $\sum_{r_i \in \mathcal{R}'} \frac{x_i}{\alpha} \phi_i(b_i) = (1 - 1/e) \sum_{r_i \in \mathcal{R}'} x_i \phi_i(b_i)$ , which is  $(1 - 1/e)$  of the optimal.

For each winner, the payment can be calculated as follows:

$$p_i = \frac{1}{x_i} \left( \sum_{j \neq i} \phi_j(b_j) x_j' - \sum_{j \neq i} \phi_j(b_j) x_j \right), \quad (14)$$

where the vector  $\mathbf{X}' = (x_j')_n$  is obtained by computing LP(2) with  $b_i = 0$ . We show that this allocation and payment mechanism results in an auction, which is truthful in expectation.

**Theorem 9.** *CATE is truthful in expectation.*

**Proof.** Let  $u_i(b_i)$  be the profit of request  $r_i$  when bidding with  $b_i$ ,  $v_i$  be the true valuation of request  $r_i$ . The expected profit of  $r_i$  is

$$\begin{aligned} E[u_i(b_i)] &= \frac{x_i}{\alpha} \left[ v_i - \frac{1}{x_i} \left( \sum_{j \neq i} \phi_j(b_j) x_j' - \sum_{j \neq i} \phi_j(b_j) x_j \right) \right] \\ &= \frac{1}{\alpha} \left[ v_i x_i + \sum_{j \neq i} \phi_j(b_j) x_j - \sum_{j \neq i} \phi_j(b_j) x_j' \right]. \end{aligned} \quad (15)$$

Since  $\sum_{j \neq i} \phi_j(b_j) x_j'$  keeps unchanged when we increase or decrease the bid of  $r_i$ ,  $E[u_i(b_i)]$  is maximized when  $b_i = v_i$ . That means the expected profit of  $r_i$  is maximized when  $r_i$  bids truthfully.  $\square$

**Theorem 10.** *Buyers will report their requested time slots and areas truthfully in CATE.*

**Proof.** When buyer  $i$  bids a larger area or more time slots than that he needs,  $\sum_{j \neq i} \phi_j(b_j) x_j'$  will keep unchanged and  $\sum_{r_j \in \mathcal{R}} \phi_j(b_j) x_j$  will never increase. According to Eq. (15),  $E[u_i(b_i)]$  keeps unchanged or decreases when buyer  $i$  misreports his need. Thus, the rational and selfish buyers will report their requested time slots and areas truthfully in CATE.  $\square$

The values of  $q(f)$  can be found by solving by the following LP(3):

$$\min \sum_{f \in \mathcal{L}} q(f),$$

subject to

$$\begin{cases} \sum_{f \in \mathcal{L}} x_i^f q(f) = \frac{x_i}{\alpha}, \forall r_i \in \mathcal{R}' \\ \sum_{f \in \mathcal{L}} q(f) \geq 1 \\ q(f) \geq 0, \forall f \in \mathcal{L}. \end{cases}$$

The dual of LP(3) is LP(4):

$$\max z + \sum_{r_i \in \mathcal{R}'} \frac{x_i}{\alpha} w_i,$$

subject to

$$\begin{cases} z + \sum_{r_i \in \mathcal{R}'} x_i^f w_i \leq 1, \forall f \in \mathcal{L} \\ z \geq 0. \end{cases}$$

We can view  $w$  in LP(4) as a valuation. If there exists an  $\alpha$ -approximation algorithm *App* that proves an integrality gap of  $\alpha$  with the optimal solution of LP(2), it has been shown in [18] that a separation oracle for LP(4) can be obtained by using Algorithm *App* with valuation  $w$ , so the ellipsoid method can be used to solve LP(4) and thus LP(3). In CATE, we choose the channel allocation algorithm DCA in the previous section as *App*. Then, we have  $\alpha = \frac{e}{e-1}$ . Since the probability of any request  $r_i$  being assigned a channel is exactly  $\frac{x_i}{\alpha}$ , we conclude that the expected total valuation of the solution of CATE is at least  $1 - 1/e$  times the total valuation of the optimal solution to IP(1). More details of CATE are shown in Algorithm 4.

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**Algorithm 4.** CATE: Truthful in Expectation Channel Allocation and Payment Calculation Mechanism

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**Input:** the set of channels  $\mathcal{S}$ , the set of requests  $\mathcal{R}$ ;

**Output:** the channel assignment vector  $\mathbf{X}$ , payment vector  $\mathbf{P}$ ;

- 1: Solve LP(2) optimally, let  $\mathbf{X}^*$  be the solution;
  - 2: Use ellipsoid algorithm and DCA on LP(4) with  $\mathbf{X}^*$  to compute the set of solutions  $\{x^f\}_{f \in \mathcal{L}}$ ;
  - 3: Solve LP(3) to compute the probability of solutions  $\{q(f)\}_{f \in \mathcal{L}}$ ;
  - 4: Choose a solution  $f$  in  $\mathcal{L}$  randomly with the probability of  $q(f)$ , let  $f^*$  be the chosen solution;
  - 5: Let  $\mathbf{X} = \mathbf{X}^{f^*}$ , where  $\mathbf{X}^{f^*}$  is the channel assignment vector of solution  $f^*$ ;
  - 6: **for**  $i = 1$  to  $n$  **do**
  - 7:     **if**  $x_i^{f^*} = 1$  **then**
  - 8:         Compute  $p_i$  by using Eq. (14);
  - 9:     **else**
  - 10:          $p_i = 0$ ;
  - 11: **return**  $(\mathbf{X}, \mathbf{P})$ ;
- 

**Theorem 11.** *CATE can be executed in polynomial time.*

**Proof.** With DCA and the optimal solution of LP(2), we apply the ellipsoid method on LP(4) to compute a set of feasible solutions of IP(1). Since the ellipsoid method takes a polynomial number of steps and DCA is also executed in polynomial time, they will return a solution set

$\mathcal{L}$  of a polynomial size. Obviously, LP(3) can also be solved in polynomial time if  $\mathcal{L}$  has a polynomial size. This completes the proof.  $\square$

## 6 FURTHER DISCUSSION

In the previous mentioned spectrum auction model, we assume that each request can only be allocated in one channel and the requested area of a buyer should be fully covered by the license areas of the allocated channel. In this section, we give a further discussion on how to relax these assumptions.

### 6.1 Requests Can Be Allocated in Different Channels

We first relax the assumption that each request can only be allocated in one channel. In fact, this relaxed model can be seen as a special case of LP(2). Moreover, we have the following theorem:

**Theorem 12.** *DCA and MDCA can also achieve the approximation factor of  $(1 - 1/e)$  in the relaxed model, where requests can be allocated in different channels.*

**Proof.** In LP(2), we have a restriction that the time assigned to a request  $r_i$  from all channels should be no longer than the required time of request  $r_i$ . Besides this, we also restrict that the time slots assigned to the request  $r_i$  from different channels should be different in the relaxed model. In other words, LP(2) describes a more general case than the relaxed model. We have proved that the total valuations of DCA and MDCA are at least  $(1 - 1/e)$  times of the total valuation of the optimal solution of LP(2). Thus, DCA and MDCA can also achieve the approximation factor of  $(1 - 1/e)$ , even if we relax the assumption of each request only being allocated in one channel.  $\square$

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#### Algorithm 5. EMDCA: The Extended Version of MDCA

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**Input:** the set of channels  $\mathcal{S}$ , the set of requests  $\mathcal{R}$ ;

**Output:** the channel assignment vector  $\mathbf{X}$ ;

- 1: Run algorithm MDCA to allocate requests in channels;
  - 2: Sort all the losers of MDCA in the descending order of their bids;
  - 3: **for** each loser  $r_i$  in the sorted list **do**
  - 4:     **if**  $r_i$  can be allocated in channels **then**
  - 5:         Allocate  $r_i$  in channels;
  - 6:     **else**
  - 7:         Set  $r_i$  as a loser;
  - 8: **return**  $\mathbf{X}$
- 

Although DCA and MDCA can be directly used in the relaxed model and provide a  $(1 - 1/e)$  performance guarantee, their performance can still be further improved by a simple modification. The details are shown in Algorithm 5. We first use MDCA to allocate requests in channels, and then we sort the losers of MDCA in the descending order of their bids. After that, we scan all of the sorted requests one by one to check whether each of them can be allocated in channels or not. When request  $r_i$  is considered, we first compute the total remaining time of all the channels, and then check whether there exists enough time to be allocated to  $r_i$ . If  $r_i$

can be allocated to channels, we scan all the channels to find the time slots which can serve  $r_i$ ; otherwise, we will drop  $r_i$ .

**Theorem 13.** *The EMDCA is bid-monotone.*

**Proof.** Since we have proved that MDCA is bid-monotone, we only need to prove that if request  $r_i$  loses in MDCA but wins in EMDCA with bid  $b_i$ , it will always win in EMDCA by bidding  $b'_i > b_i$ . Obviously, the sequence of  $r_i$  in the loser list of MDCA will not decrease if  $r_i$  remains a loser in MDCA with bid  $b'_i$ . Moreover, the requests, which overlap with  $r_i$  and lose in MDCA when  $r_i$  bids  $b_i$ , will still lose when  $r_i$  bids  $b'_i > b_i$ . Thus, there are also enough time slots for  $r_i$  when he bids  $b'_i > b_i$  if he can win in EMDCA with bid  $b_i$ .  $\square$

Since EMDCA is bid-monotone, we can use the binary search to find the critical value for each winner as its payment. Then, we get a truthful auction mechanism.

### 6.2 Each Buyer Has Multiple Requests

Now, we relax the assumption that the requested area of a buyer should be fully covered by the license areas of the allocated channel. When a buyer's requested area overlaps with the license areas of multiple channels, we allow this buyer to submit multiple requests in one auction, each of which is fully covered by the license areas of one allocated channel. There are two possible model: The first one is that all the requests of a winning buyer should be fully satisfied. This case is also known as the combinatorial auction model [19], [21], and it has been proved that  $\sqrt{m}$  is tight for combinatorial auction. Thus, we only consider the second case that a buyer can be satisfied by part of his requests.

Since the demand of a buyer can be partly satisfied, we view a multi-request buyer as multiple virtual buyers. Each virtual buyer has one request. Then, we can directly solve this problem by using the approximation algorithms proposed in previous sections. Moreover, we can prove that the auction mechanisms design in the previous sections can still preserve the truthfulness under this model.

**Theorem 14.** *MDCA and CATE can preserve the truthfulness under the multi-requests model.*

**Proof.** In the multi-requests model, the requests from each buyer will not conflict with each other under different non-overlapping license areas of channels. Thus, they will compete for different channels and will not affect others' auction results and payments. In other words, buyers cannot misreport a portion of his requests to benefit the remaining requests. Since the requests which belong to the same buyer cannot improve their utilities through collusion, our auction mechanism design in the previous sections can still preserve the truthfulness under this model.  $\square$

## 7 SIMULATION RESULTS

We conduct extensive simulations to evaluate the performance of the proposed auction mechanisms. Below we first introduce the simulation settings and performance metrics, and discuss the algorithms under comparison. We then present the results under various settings.

## 7.1 Simulation Settings and Metrics

In the simulations, we assume that there is only one seller, who subleases the usage of three channels in the spectrum market. The auction period  $T$  is six days. The license area of each candidate spectrum follows the disk model, and the radius of each license area is randomly selected from 40 to 70 units of distance (where a unit may be a kilometer or any other chosen measure). All the buyers are randomly distributed within a fixed area of  $100 \times 100$  square units. Our simulations use the point model as the buyer request model. For conciseness, we do not include the other buyer request model, i.e., the area model, because the two models only differ in the definition of conflict, and they will lead to the same performance analysis results. We also assume that all the buyers' bid values are uniformly, exponentially or Gaussian distributed in  $(0, 1]$ , and the time duration  $t_i$  for each request  $r_i$  is randomly generated from one to three days. All simulation results are the average of 50 runs.

We adopt the widely used performance metrics, including social efficiency ratio, revenue ratio, and spectrum utilization ratio. The social efficiency ratio of an algorithm is the ratio of the social efficiency of this algorithm and that of the optimal solution. The revenue ratio is the ratio of the total payments from winners and the optimal social efficiency, which is actually an upper bound of the revenue. The spectrum utilization ratio is the ratio of the combined time allocated to all winners and the total time available by all the channels for allocation.

## 7.2 Algorithms under Comparison

In the evaluation, we compare the performance of the proposed algorithms, including DCA, MDCA, and CATE, as well as the best that the existing auction mechanisms can achieve. Unlike our algorithms that take both spatial and temporal reuse into account, the previous spectrum auction mechanisms only consider either spatial reuse or temporal reuse separately. (Moreover, they are generally designed for different scenarios and settings.) Hence, we use an optimal spectrum auction algorithm only with spatial reuse and an optimal spectrum auction algorithm only with temporal reuse for comparison under our simulation settings. The two algorithms are named OPT\_S and OPT\_T, respectively.

OPT\_T is the optimal solution of the channel allocation problem when the buyers can only share a single channel in temporal domain, which is given by the following IP(5):

$$\max \sum_{s_j \in \mathcal{S}} \sum_{r_i \in \mathcal{R}'} \phi_i(b_i) x_{i,j},$$

subject to

$$\begin{cases} \sum_{s_j \in \mathcal{S}} x_{i,j} \leq 1, \forall r_i \in \mathcal{R}' \\ x_{i,j}^l \leq c_{i,j}, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}', \forall l \\ \sum_{k \neq i} x_{k,j}^l y_{i,k,j} + x_{i,j}^l \leq 1, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}', \forall l \\ \sum_{l=a_i}^{d_i} x_{i,j}^l t_j^l = t_i x_{i,j}, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}' \\ x_{i,j} \in \{0, 1\}, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}' \\ x_{i,j}^l \in \{0, 1\}, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}', \forall l. \end{cases}$$

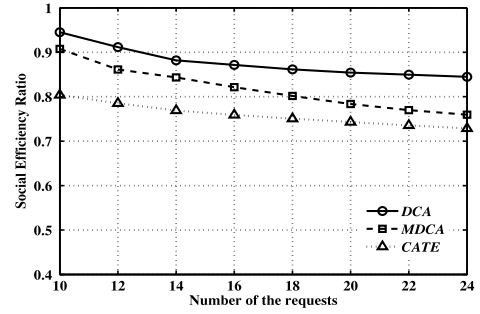


Fig. 3. Social efficiency under uniform distribution.

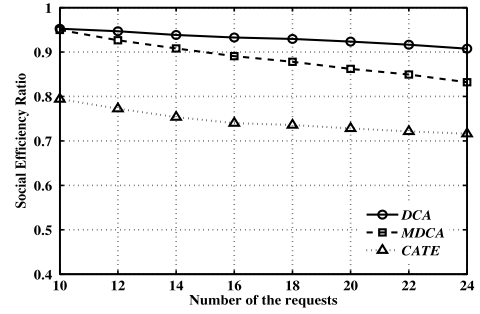


Fig. 4. Social efficiency under exponential distribution.

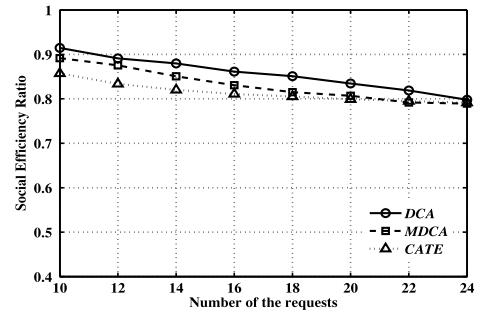


Fig. 5. Social efficiency under Gaussian distribution.

Here,  $y'_{i,k,j} = 1$  if requests  $i$  and  $k$  can share channel  $j$  in temporal domain, and  $y'_{i,k,j} = 0$  otherwise.

Similarly, OPT\_S is the optimal solution of the channel allocation problem when the buyers can only share a single channel in spatial domain, which is given by IP(6):

$$\max \sum_{s_j \in \mathcal{S}} \sum_{r_i \in \mathcal{R}'} \phi_i(b_i) x_{i,j},$$

subject to

$$\begin{cases} \sum_{s_j \in \mathcal{S}} x_{i,j} \leq 1, \forall r_i \in \mathcal{R}' \\ x_{i,j}^l \leq c_{i,j}, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}' \\ \sum_{k \neq i} x_{k,j} y_{i,k,j} + x_{i,j} \leq 1, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}' \\ x_{i,j} \in \{0, 1\}, \forall s_j \in \mathcal{S}, \forall r_i \in \mathcal{R}'. \end{cases}$$

Here,  $y''_{i,k,j} = 1$  if requests  $i$  and  $k$  can share channel  $j$  in spatial domain, and  $y''_{i,k,j} = 0$  otherwise.

## 7.3 Performance Analysis on Social Efficiency

We first evaluate the social efficiency performance of the proposed algorithms DCA, MDCA and CATE. The interference radius of each channel is set to be 30. For each type



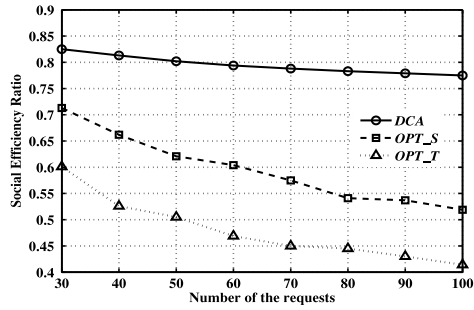


Fig. 6. Social efficiency ratio comparison.

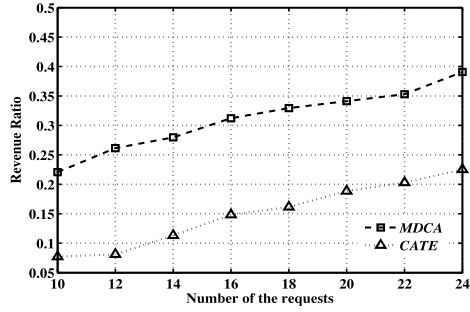


Fig. 7. Revenue ratio under uniform distribution.

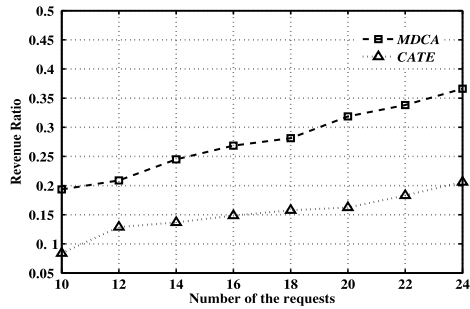


Fig. 8. Revenue ratio under exponential distribution.

of bid distribution (i.e., uniform, exponential and Gaussian), we record the social efficiency ratio of the three algorithms, with respect to the number of requests, shown in Figs. 3, 4, 5. As expected, the social efficiency ratios of DCA and MDCA are better than that of CATE. This is because DCA and MDCA always result in a solution whose value is larger than  $1 - 1/e$  times of the optimal one, while the solution of CATE does not have such a performance guarantee. The simulation results of all three algorithms are much better than the theoretical bound we have derived in the previous sections. Even the result of CATE is larger than 70 percent of the optimal solution.

From Figs. 3, 4, 5, we can also see that the social efficiency ratio decreases slightly as we increase the number of requests. When there are only a few requests, most requests can be allocated in channels without conflict, and in this case the three algorithms perform almost as well as the optimal auction mechanism. However, as the number of requests increases, the competition among the requests also increases and the performance of DCA, MDCA and CATE decreases gradually.

Next, we compare the social efficiency performance of the DCA algorithm with OPT\_S and OPT\_T. In this evaluation, the uniform bid distribution is adopted, and the

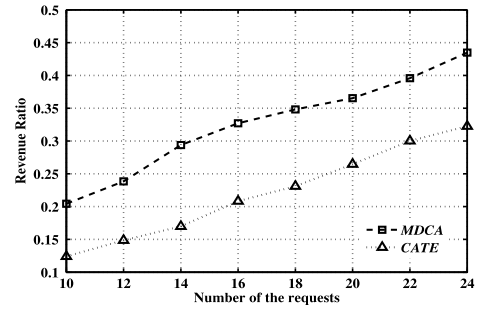


Fig. 9. Revenue ratio under gaussian distribution.

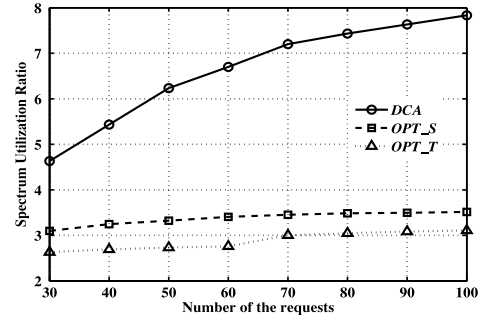


Fig. 10. Spectrum utilization ratio.

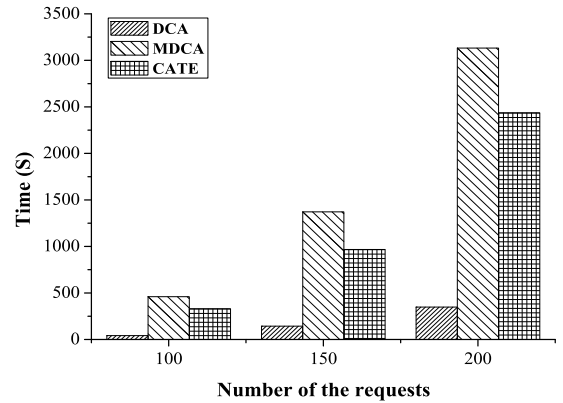


Fig. 11. Run time comparison over different mechanisms.

interference radius is selected from 12 to 15. Fig. 6 plots the social efficiency ratio of DCA, OPT\_S, and OPT\_T, which shows that the performance of DCA is much better than OPT\_S and OPT\_T. It means that not only can DCA increase the channel utilization, but also it can improve the social efficiency. Moreover, Fig. 6 shows that the improvement of DCA over OPT\_S and OPT\_T widens with more requests.

#### 7.4 Performance Analysis on Revenue

Next we evaluate the revenue performance of the proposed algorithms, including MDCA and CATE. Here, we ignore the DCA algorithm. This is because we cannot prove that DCA is bid monotone, and we cannot compute the critical payment for each winner either. The revenue ratio results of MDCA and CATE are shown in Figs. 7, 8 and 9. We can see that the revenue ratio of the **seller** increases along with the number of requests, when the reservation price stays the same. That is because the payment of each winner in our auction mechanisms is a critical value, which becomes larger along with an increasing level of competition among requests.

## 7.5 Other Performance Analysis

Finally, we study the spectrum utilization efficiencies of DCA, OPT\_S and OPT\_T. As shown in Fig. 10, the spectrum utilization efficiencies of the proposed algorithms are much better than OPT\_S and OPT\_T, which allow spectrum reuse in either spatial or temporal domain. In addition, the spectrum utilization ratios of OPT\_S and OPT\_T become flat when the number of requests is larger than 70. In comparison, the spectrum utilization ratio of DCA keeps increasing along with the number of requests. This indicates that when the request number reaches a high level, they can still be satisfied by DCA with the consideration of both spatial and temporal reuses at the same time.

We also test the computation overhead of our proposed algorithms. The hardware/software platform is a laptop with *Intel(R) Core(TM) i5 2.4 GHz, 4.0 GB RAM and Windows 7 64-bit*. Note that the optimal channel allocation problem in our model is an NP hard problem. For the purpose of comparison, we implement the optimal channel allocation scheme, denoted by OPT; when there are 15 requests, the running time of OPT is about 2,652 s. The running times of DCA, MDCA and CATE are shown in Fig. 11. DCA only needs about 348 s, even when there are 200 requests, which is much less than OPT. The upper bound of the running time of MDCA is  $2MN$  times of the running time of DCA, where  $M$  is the number of channels and  $N$  is the number of requests. However, the actual running time of MDCA is 3,132 s, when there are 200 requests, which is much less than what the upper bound suggests. We have claimed that CATE is more efficient than MDCA in our theoretical analysis. This is because CATE is proved to be a polynomial time algorithm, whereas the running time of MDCA mainly depends on the ratio of the max bid among requests to the step size of bids in the process of payment calculation, which is not theoretically polynomial. In our simulation, we set all the buyers' bid values to be distributed in  $(0, 1]$ , and set the step size of bids to be 0.000001. As shown in Fig. 11, the simulation results corroborate our theoretical analysis. If the ratio of the max bid among requests to the step size of bids is large enough, CATE will perform much better than MDCA.

## 8 LITERATURE REVIEWS

Auction theory, regarded as a subfield of economics and game theory, serves as an efficient and fair way to distribute scarce resources among competing users. In recent years, various auction models have been successfully designed in the communication and networking field [16], [27], [36], [37]. For instance, Yang et al. proposed TASC [36], a double auction scheme for the cooperative communication scenario. A similar work for cooperative communications was proposed to maximize each user's profit function with the knowledge of others' previous bids [16].

There are also many state-of-the-art auction mechanisms that have been extensively studied in the scope of spectrum allocation [10], [15], [24]. They mainly cope with the dynamic spectrum access problem from various perspectives by using different optimization goals, such as maximizing the total profit or minimizing the spectrum interference.

Truthfulness (or strategyproofness) is considered as one of the most critical factors in the design of auction mechanism. Although a large number of auction mechanisms have been designed to achieve economical robustness (e.g. [2], [7], [20], [25]), when these mechanisms are directly applied to spectrum auctions, they will lose the truthful property, due to some constraints, such as spatial and temporal reuse of spectrum. Meanwhile, some well-known auction mechanisms (such as VCG [5], [12], [25]) will also lose truthfulness when applied to suboptimal algorithms. Therefore, these auction mechanisms are not suitable for spectrum auction.

In recent years, some studies investigate the truthful auction model with spectrum spatial reuse [9], [11], [13], [17], [28], [29], [30], [38], [39], [40], [41]. They do not consider the temporal demands from buyers. Truthfulness is first introduced in [39] for spectrum auction, where the spatial reuse is considered. Maximizing the revenue for auctioneers are studied in [1] and [17]. A combinatorial auction model for the heterogeneous channel redistribution is proposed in [38], achieving both strategy-proofness and approximately efficient social welfare. Trade-off between fairness and maximizing social welfare is investigated in [11] with a truthful spectrum auction model. Zhou and Zheng [40] first takes the extended McAfee double auction model into spectrum allocation to achieve the economic robustness.

On the other hand, spectrum is a local resource. It is usually traded within its license region through a secondary market. Thus, *District* mechanism [28] first takes the spectrum locality into consideration and proposes an economically robust double auction method. Feng et al. [9] proposes a truthful double auction model for heterogeneous spectrum trading with the consideration of spectrum reusability and spectrum locality. As another line of spectrum reuse, [6], [26], [32], [33], [34], [35] study the spectrum allocation with an online model. The temporal reuse is adopted in these online-model studies. In addition, Xu et al. [34] propose SALSA for online spectrum admission, which can achieve a constant approximation compared to the offline VCG auction in both social efficiency and revenue efficiency. A truthful online double spectrum auction mechanism TODA is presented by Wang et al. [26] to achieve economic-robustness.

However, the combination of spectrum locality and temporal reuse has not been considered in these previous studies. Although Dong et al. [8] tackles spectrum auction by introducing a combinatorial auction model, which achieves time-frequency flexibility, they do not consider spatial reuse and spectrum locality property in their work. In comparison, this paper generalizes all of the above challenges in the auction design.

## 9 CONCLUSION

In this paper, we have studied the problem of spectrum auction where channels can be reused both spatially and temporally. We have designed a general truthful spectrum auction framework which can maximize social efficiency or revenue. While the optimal channel allocation is NP-hard under our model, we have developed a series of near-optimal spectrum auction mechanisms with  $(1 - 1/e)$  performance guarantee.

Some interesting questions are left for future research. First, we plan to relax the request model from fixed time intervals studied in this paper to a more general one by allowing the time duration  $t_i$  to be smaller than the difference between the beginning time  $a_i$  and the ending time  $d_i$  of each request. Second, we plan to further allow each request to have multiple time intervals. Third, we plan to design truthful mechanisms with good performance guarantee for online auctions, where requests are processed as they arrive.

## ACKNOWLEDGMENTS

Yu-e Sun is the corresponding author. The research of authors is partially supported by the US National Science Foundation (NSF) CNS-0832120, NSF CNS-1035894, NSF ECCS-1247944, NSF ECCS-1343306, National Natural Science Foundation of China under Grant No. U1301256, No. 61170216, No. 61228202, No. 61303206, No. 61202028. Any opinions, findings, conclusions, or recommendations expressed in this paper are those of author(s) and do not necessarily reflect the views of the funding agencies (NSF, and NSFC). The preliminary version of this paper appeared in MobiHoc 2013 [14].

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