

Overflow Handling

- An overflow occurs when the home bucket for a new pair (**key, element**) is full.
- We may handle overflows by:
 - Search the hash table in some systematic fashion for a bucket that is not full.
 - Linear probing (linear open addressing).
 - Quadratic probing.
 - Random probing.
 - Eliminate overflows by permitting each bucket to keep a list of all pairs for which it is the home bucket.
 - Array linear list.
 - Chain.

Linear Probing – Get And Put

- divisor** = **b** (number of buckets) = 17.
- Home bucket** = **key % 17**.

0	4	8	12	16
34	0	45		
		6	23	7
			28	12
			29	11
			30	33

- Put in pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45

Linear Probing – Remove

0	4	8	12	16
34	0	45		
		6	23	7
			28	12
			29	11
			30	33

- remove(0)**

0	4	8	12	16
34		45		
		6	23	7
			28	12
			29	11
			30	33

- Search cluster for pair (if any) to fill vacated bucket.

0	4	8	12	16
34	45			
		6	23	7
			28	12
			29	11
			30	33

Linear Probing – remove(34)

0	4	8	12	16
34	0	45		
		6	23	7
			28	12
			29	11
			30	33

0	4	8	12	16
	0	45		
		6	23	7
			28	12
			29	11
			30	33

- Search cluster for pair (if any) to fill vacated bucket.

0	4	8	12	16
0		45		
		6	23	7
			28	12
			29	11
			30	33

0	4	8	12	16
0	45			
		6	23	7
			28	12
			29	11
			30	33

Linear Probing – remove(29)

0	4	8	12	16
34	0	45		
		6	23	7
			28	12
			29	11
			30	33

0	4	8	12	16
34	0	45		
		6	23	7
			28	12
				11
				30
				33

- Search cluster for pair (if any) to fill vacated bucket.

0	4	8	12	16
34	0	45		
		6	23	7
			28	12
				11
				30
				33

0	4	8	12	16
34	0	45		
		6	23	7
			28	12
				11
				30
				33

0	4	8	12	16
34	0			
		6	23	7
			28	12
				11
				30
				45
				33

Performance Of Linear Probing



0	4	8	12	16
34	0	45		
		6	23	7
			28	12
			29	11
			30	33

- Worst-case get/put/remove time is **Theta(n)**, where **n** is the number of pairs in the table.
- This happens when all pairs are in the same cluster.

Expected Performance



0	4	8	12	16
34	0	45		
		6	23	7
			28	12
			29	11
			30	33

- α = loading density = (number of pairs)/b.
 - $\alpha = 12/17$.
- S_n = expected number of buckets examined in a successful search when n is large
- U_n = expected number of buckets examined in a unsuccessful search when n is large
- Time to put and remove governed by U_n .

Expected Performance



- $S_n \sim \frac{1}{2}(1 + 1/(1 - \alpha))$
- $U_n \sim \frac{1}{2}(1 + 1/(1 - \alpha)^2)$
- Note that $0 \leq \alpha \leq 1$.

α	S_n	U_n
0.50	1.5	2.5
0.75	2.5	8.5
0.90	5.5	50.5

$\alpha \leq 0.75$ is recommended.

Hash Table Design

- Performance requirements are given, determine maximum permissible loading density.
- We want a successful search to make no more than 10 compares (expected).
 - $S_n \sim \frac{1}{2}(1 + 1/(1 - \alpha))$
 - $\alpha \leq 18/19$
- We want an unsuccessful search to make no more than 13 compares (expected).
 - $U_n \sim \frac{1}{2}(1 + 1/(1 - \alpha)^2)$
 - $\alpha \leq 4/5$
- So $\alpha \leq \min\{18/19, 4/5\} = 4/5$.

Hash Table Design

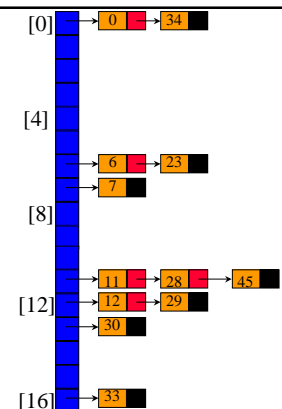
- Dynamic resizing of table.
 - Whenever loading density exceeds threshold (4/5 in our example), rehash into a table of approximately twice the current size.
- Fixed table size.
 - Know maximum number of pairs.
 - No more than 1000 pairs.
 - Loading density $\leq 4/5 \Rightarrow b \geq 5/4 * 1000 = 1250$.
 - Pick b (equal to divisor) to be a prime number or an odd number with no prime divisors smaller than 20.

Linear List Of Synonyms

- Each bucket keeps a linear list of all pairs for which it is the home bucket.
- The linear list may or may not be sorted by key.
- The linear list may be an array linear list or a chain.

Sorted Chains

- Put in pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45
- Home bucket = key % 17.



Expected Performance



- Note that $\alpha \geq 0$.
- Expected chain length is α .
- $S_n \sim 1 + \alpha/2$.
- $U_n \leq \alpha$, when $\alpha < 1$.
- $U_n \sim 1 + \alpha/2$, when $\alpha \geq 1$.

java.util.Hashtable



- Unsorted chains.
- Default initial $b = \text{divisor} = 101$
- Default $\alpha \leq 0.75$
- When loading density exceeds max permissible density, rehash with $\text{newB} = 2b+1$.