CIS6930: PCPs and Inapproximability - Homework 2

Due at the **beginning** of the lecture on **11-24-2009**.
No late assignment will be accepted.

Do the following 4 required problems, 10 pts each:

Problem 1. Using the gap-preserving reduction form Label Cover, show that for every $\epsilon > 0$, there is no $(1-\epsilon)\ln n$ approximation algorithm exists for the set cover problem unless $NP \subset DTIME(n^{O(\log\log n)}$where n is the size of the universe.

Problem 2. Use the special graph to show Problem 2 (Saving Random Bits). [Look at the lecture note Expanders.pdf page 3 and 4.]

Problem 3. Prove Theorem 1 in the lecture note. (That is: For all $c > 0$, there exists a constant $d_0 > 0$ and $n_0 > 0$ such that an (n, d, c)-edge expander graph exits for all $d \geq d_0$ and $n \geq n_0$.)

Problem 4. Let G be a d-regular graph and let $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ be the eigenvalues as mentioned in the lecture note. Show the following:

- $\lambda_1 = d$ and the corresponding eigenvector is $\mathbf{x}_1 = (\mathbf{1}/\sqrt{n})^T = (1/\sqrt{n}, \ldots, 1/\sqrt{n})^T$
- The graph is connected iff $\lambda_1 > \lambda_2$
- The graph is bipartite iff $\lambda_1 = -\lambda_n$