CIS6930: PCPs and Inapproximability - Homework 1

Due at the **beginning** of the lecture on **10-13-2009**.
No late assignment will be accepted.

Do the following 6 required problems. Each problem is 10 pts.

Problem 1. Give a gap preserving reduction from MAX-3SAT(29) to MAX-3SAT(5) with appropriate parameters to show the hardness of the latter problem.

Problem 2. Give a gap preserving reduction from MAX-3SAT(d) to MAX-E3SAT(Ed) (each variable appears exactly d times) to show the hardness of the latter problem, that is, GAP-MAX-E3SAT(Ed)$_{1,\rho}$ is NP-hard for a constant $0 < \rho < 1$

Problem 3. Prove that $\mathbf{NP} = \mathbf{PCP}_{1,1/n}[\log n, \log n]$

Problem 4. Show that if there exists an $\epsilon > 0$ for which there is a $(1+2/e-\epsilon)$-approximation algorithm for the metric k-Median problem, then $NP \subseteq DTIME(n^{O(\log\log n)})$.

Problem 5. In the class, we mentioned that $\forall \rho > 0$, GAP-CLIQUE$_{1,\rho}$ is NP-hard by running PCP verifier k times. Now, prove that theorem ($\forall \rho > 0$, GAP-CLIQUE$_{1,\rho}$ is NP-hard) using the powering graph (similar to the one we used to show the hardness of approximation of Independent Set). That is, for a graph $G = (V, E)$ and an integer $k \geq 2$, you need to define the k^{th} power of G, $G^k = (V', E')$ such that $\omega(G^k) = \omega(G)^k$ where $\omega(.)$ defines the size of largest cliques in (.). From there, prove the theorem.

Problem 6. Show that there exits a $\delta > 0$, GAP-CLIQUE$_{1,n^{-\delta}}$ is NP-hard. As hinted in the lecture note, you may want to choose k to be supper-constant, then the verifier needs a total of $O(k \log n)$random bits, then the size of G becomes supper-polynomial. Try to show that we can run the verifier k times by using only $O(\log n) + O(k)$ (rather than $O(k \log n)$)