# CIS6930: PCPs and Inapproximability - Final Exam

Due at 4pm on **12-08-2009** via email. As usual, I will NOT accept any late submission.

This is the final exam. Therefore, **no** collaboration and discussion between students is allowed. **No** references except lecture notes is accepted.

Do the following 5 required problems, 10 pts each:

**Problem 1**. Prove that it is NP-hard to approximate the Edge Disjoint Paths (EDP) problem within a factor of $m^{1/2-\epsilon}$ for any $\epsilon > 0$ where the EDP problem is defined as follows:

**Definition 1** *Given a directed graph $G = (V, E)$ with $m = |E|$ and source-sink pairs $(s_i, t_i)$ for $i = 1, \ldots, t$, find the maximum number of edge disjoint paths (paths that do not share edges) to connect these source-sink pairs (that is, our goal is to connect as many pairs as possible using edge disjoint paths)*

**Problem 2.** Prove that there is no $(2/3+\epsilon)$-approximation for MAX-3MAJ unless P = NP where MAX-3MAJ is defined as follows:

**Definition 2** *An instance of MAX-3MAJ is a collection of $m$ clauses in which each clause $C_i$ is of the form $MAJ(x_{i1}, x_{i2}, x_{i3})$ where $z_i$ is a boolean variable and $MAJ$ is the majority function (the majority of its three literals' values is 1). The problem asks to find a truth assignment for all variables so as to maximize the number of satisfied clauses.*

**Problem 3**. Prove the following:

Let $B_0, B_1, \cdots, B_t$ be subsets of $V$ such that $\beta_i = |B_i|/n$. Define $(B, t)$ to be the event that a random walk $(v_0, v_1, \cdots, v_t)$ has the property that $\forall i, \quad v_i \in B_i$, we have:

$$Pr[(B,t)] \leq \prod_{i=0}^{t-1} (\sqrt{\beta_i \beta_{i+1}} + \alpha)$$

**Problem 4**. Prove the following:

Let $G = (V, E)$ be an $(n, d, \alpha)$-graph (as defined in class), then for every $S, T \subseteq V$, we have:

$$\left| \frac{d|S||T|}{n} - E(S,T) \right| \leq \alpha d \sqrt{|S||T|}$$

**Problem 5**. Find the spectrum of a $d$-regular tree of height $h$ where $d$ and $h$ are given positive integers.